Revista Internacional de Ciências,
Rio de Janeiro, v. 09, n. 03, p. 16 - 31, set-dez 2019
http://www.e-publicacoes.uerj.br/ojs/index.php/ric

e-ISSN 2316-7041

DOI: 10.12957/ric.2019. 42656

Development of an Integrated Metrô-Bus Transportation System Using the Mathematica Software

Rafael Cabidolusso Lavalle Heilbron[1]*; Corbiniano Silva*[2]*; Paulo Fernando Lavalle Heilbron Filho*[3]*; Gleide Borges Moraes Lacerda*[2]

rafaelheilbron@hotmail.com

1. Catholic University of Rio de Janeiro, Gávea - RJ. 2. Federal University of Rio de Janeiro, Ilha do Fundão - RJ. 3. Brazilian Nuclear Energy Commission, Botafogo - RJ.

Histórico do Artigo:
Recebido em: 12 de maio de 2019 Aceito em: 18 de novembro de 2019 Publicado em: 28 de dezembro de 2019

Abstract: The article shows the application of a program developed in the computational platform Mathematica, used to organize the urban transport that involves a conjugated metro-bus lines system for collective use. Three different bus lines were considered, connecting several neighborhoods. The results show the number of buses required, estimated times, arrival and departure times for three bus lines, from a hypothetical location, based on simulated data, showing the potential of numerical modeling to develop an integrated system with the subway. The program can be easily extended, allowing the inclusion of more bus lines and immediate results for any desired change / insertion of data, allowing a better and faster evaluation of the problem. The study and its results present an important tool of support and decision making to subsidize the urban transport planning of medium and large cities.
Keywords: Urban Transport, Integrated metro-bus system, Computational modeling, Mathematica.

Desenvolvimento de um Sistema de Transporte Integrado Metrô-Ônibus Utilizando o Software Mathematica

Resumo: O artigo mostra a aplicação de um programa desenvolvido na plataforma computacional Mathematica, utilizada para organizar o transporte urbano que envolve um sistema de metro-linhas de ônibus conjugadas para uso coletivo. Três diferentes linhas de ônibus foram consideradas, conectando diversos bairros. Os resultados mostram o número de ônibus necessários, tempos estimados, horas de chegada e saída para três linhas de ônibus, a partir de uma localização hipotética, com base em dados simulados, mostrando o potencial da modelagem numérica para desenvolver um sistema integrado com o metrô. O programa pode ser facilmente estendido, admitindo a inclusão de mais linhas de ônibus e resultados imediatos para qualquer alteração/inserção de dados desejada, permitindo uma avaliação melhor e rápida do problema. O estudo e seus resultados apresentam uma ferramenta importante de suporte e tomada de decisão para subsidiar o planejamento do transporte urbano de médias e grandes cidade.
Palavras-chave: Transporte urbano, Sistema integrado ônibus-metrô, Modelagem computacional, Mathematica.

Desarrollo de um Sistema de Transporte Integrado Metró-Autobús Utilizando el Software Mathematica

Resumen: El artículo muestra la aplicación de un programa desarrollado en la plataforma computacional Mathematica, utilizada para organizar el transporte urbano que envuelve un sistema de líneas de autobús conjugadas para uso colectivo. Tres diferentes líneas de autobuses fueron consideradas, conectando diversos barrios. Los resultados muestran el número de autobuses necesarios, tiempos estimados, horas de llegada y salida para tres líneas de autobús, desde una ubicación hipotética, con base en datos simulados, mostrando el potencial del modelado numérico para desarrollar un sistema integrado con el metro. El programa puede ser fácilmente extendido, admitiendo la inclusión de más líneas de autobús y resultados inmediatos para cualquier cambio / inserción de datos deseada, permitiendo una evaluación mejor y rápida del problema. El estudio y sus resultados presentan una herramienta importante de soporte y toma de decisión para subsidiar la planificación del transporte urbano de medianas y grandes ciudades.
Palabras clave: Transporte urbano; Sistema integrado autobús-metro; Modelado computacional; Mathematica.

INTRODUCTION

The circulation and movement of the population to work, schools, health, leisure, among other purposes, is a necessity of the human being, where collective transportation plays a fundamental role. However, with population growth there is an increase in the demand for transportation, triggering the expansion of the fleet of vehicles, reflecting the intensification of agglomerations and congestion in urban areas complicating the logistics of the road system becoming more complex.

As just pointed out by Carvalho (2016) forty years ago, the Brazilian population lived mostly in rural areas, without much demand for mass transportation in the few existing urban agglomerations. Today, about 85% of the population lives in urban centers, and there are 36 cities with more than 500,000 inhabitants in the Brazilian urban network, as well as 40 established metropolitan regions, where more than 80 million Brazilians live (about 45% of the population).

According to IPEA (2011) it is worth mentioning the challenges faced by the rulers in the implementation of popular housing policies in regions far from the city important centers, that causes the immobility of the poorest and the formation of dormitory cities, putting increasing pressure on the global cost of transportation. Between the 2000 and 2010 censuses, the peripheral municipalities of the main Brazilian metropolitan regions had a much higher population growth than the central municipalities.

In medium and large cities, with increasing distances and times of travel on journeys, especially between peripheral areas and central regions, the level of bus parking, waiting time by the passengers and the logistics of the city show the low-level service provided to the population.

According to Pena (2017) the main cause of urban mobility problems in Brazil is related to the increased use of individual transport over the use of public transport, although the latter also encounter difficulties with overcrowding. This increased use of vehicles such as cars and motorcycles is due to:

a) The poor quality of public transportation in Brazil;

b) The increase in the average income of Brazilians in recent years;

c) The reduction of taxes by the Federal Government on industrialized products (which includes cars);

d) The granting of more consumer credit;

e) The historical heritage of the country's road policy.

Also according to Pena (2017) between 2002 and 2012, according to data from the Observatory of the Metropolis, while the Brazilian population increased by 12.2%, the number of vehicles registered a growth of 138.6%. There are cities in the country that present a less than two inhabitants for each car present, which makes almost all measures impossible to guarantee a more efficient transport system.

This problem highlights the need for improvements in urban transport systems, whose progress in quality results from the integration of their systems, where indicators of accessibility, frequency of service, travel time, stocking, reliability, (Ferraz & Torres, 2001). In addition, the use of public transport is a key factor in the quality of public transportation.

In this perspective, the adoption of integrated bus corridors to other transport systems (train, subway, light rail transport) and its better management in the logistics of cities contributes to the improvement of the quality of public transport.

One of the most important tasks relating to urban transport is the development and correct planning of a complete integrated system connecting, for example, metro and buses routes (Vuchic, 2005; 2007).

Usually, big cities have an integrated metro-bus complex system that has to be developed based on many studies and researches especially in the peak time traffic in order to have an optimized system. Almasi *et al.* (2016) based their integrated model based on geometric series but with focus on studies of the time, that a bus would stayed at the stop to load/unload other passengers in order to optimize the metro-bus system. Oguchi *et al.* (2017) on the other hand studied the inefficiencies that could result from a poor feeder service that, eventually, can make an urban mass transit system unsustainable. Thilakaratne *et al.* (2016) and Geurs *et al.* (2016) focused their modeling on the demand/supply interactions due to implementation of new technologies or changes in mode of service, especially, in regions where more than one transport feeder mode exists. The first author based his studies on a bus and van integrated metro system. The second analyzed the possibility of a bicycle – metro integration system.

Others authors based their work on modeling the effects of a station disruption, an abnormal operational situation when the entrance or exit gates of a metro station have to be closed for a certain of time due to an unexpected incident, establishing a model to solve the metro station disruption problem by providing optimal additional bus-bridging services in that case. There are many studies around the world involving integrated transport systems in order to improve their quality such those from:

• Chakrabarti & Giuliano (2015) that used data from the Los Angeles metro-bus system to analyze the variation in intersections between the lines and to find the reliability of the service, being decisive to promote the sponsorship of transit systems with fixed schedules and routes, whose improvements in reliability can lead to productivity gains for traffic agencies;

• Chakrabarti (2015) that evaluated whether the reliability of the transportation service determines the number of transit passengers in the Los Angeles metro system, and whether its improvements helped to promote the number of transit passengers during the peak period of the system;

• Chen & Tseng (2016) that examined the metro-bus intermodal system and its use by passengers, influenced by different amounts of information, which evaluated the managerial implications for metro-bus system agencies and applications in intermodal transfer services to other industries carriage;

• Cereja *et al.* (2012) that assessed the need for good intermodal connections in transport systems in Bangkok, Thailand, with actions that can improve the connections between metro and buses, suggesting improvements in intermodal bus-metro transfers, including security conditions against crime and the distance between metro exits and bus stops;

• Xin *et al.* (2016) that evaluated the multiperiod time optimization for metro networks, analyzing the problem of the optimization of train schedules for metropolitan transit networks to improve the performance of the synchronization of transfer between different railway lines, especially in the problem of optimization of schedules in the transition period (from peak to off peak hours or vice versa), where train advance changes and passenger travel demand vary significantly;

• Hidalgo (2009) that analyzed the interconnected transit system of São Paulo (Brazil), which optimized bus routes and services through the use of advanced technologies for tariff integration, fleet renewal and requirements for the provision of transit services and support infrastructure for buses (priority and exclusive tracks, bus stops, integration terminals and information systems and user control). The time integration scheme has changed the way passengers select a combination of services and has resulted in time and travel savings. A decrease in cars was identified, associated with the main complaints about high levels of pollution, congestion and long waiting periods and trips;

• Jin *et al.* (2014) that evaluated Singapore's transport system via metro-bus integration, whose results show that the resilience of the metropolitan network to disruptions can be significantly improved from localized integration with public bus services;

• Kepaptsoglou & Karlaftis (2009) that analyzed the metro network and its unexpected operational disruptions, with the rapid and efficient replacement of services needed to accommodate passengers, including the widely used "bridge" practice of metro stations using bus services, being proposed a methodological structure to plan and design an efficient network of bus bridges, bus route project and decision support system;

• Königsberg (2008) that presented an algorithm applied to the metro-bus public transport system in Mexico City to calculate a generalized model of regular reducible matrices over max-plus algebra, whose application allowed to obtain a schedule for the integrated transport system;

• Kumar *et al.* (2011) that assessed the metro-bus multimodal transport system in Nova Dheli (India), analyzing the integration and exchange of the system and the perceptions of safety of the exchange of passengers for the implementation of general and specific safety measures, elements that should contribute to make public transport systems safer;

• Li *et al.* (2011) that presented a statistical approach to predict the arrival time of public bus transportation, based on the traffic information management system, where several factors affect the travel time of buses (departure time, business day, location of buses, number of connections, number of intersections, passenger demand at each stop, and urban network traffic status to describe bus arrival times) were considered;

• Mata *et al.* (2011) that developed a mobile assistant application to locally locate and orient passengers of a bus-metro system in Mexico City, which assists blind or visually impaired passengers from an affordable and low-cost combination of mobile devices, GPS and compass to provide audible orientated functionalities and interfaces that indicate to a given user their location within the bus-metro system, with appropriate guidance instructions leading to boarding gates at that station, thanks to a developed guidance algorithm;

• Pekel & Kara (2016) that established in Istanbul (Turkey) a fleet schedule, based on simulation in the metro-bus system to meet passenger satisfaction, whose shipment and landings distributions are determined in relation to the stochastic process and a model of discreet event built to determine at which point a bus embarks and disembarks from the metro-bus. The results prove that the simulation-based fleet scheduling can be successfully applied to the metro system;

• Seriani & Fernández (2015) that proposed directives to the planning of interchanges of the metro-bus system, through the application of a model of pedestrian microsimulation in Santiago of Chile;

• Song *et al.* (2012) that developed and applied a 'scientific approach' to improve the integration of metro-bus networks in Beijing, China, whose proposed approach can provide meaningful information for service planning in multimodal corridors in Chinese cities, where metropolitan networks are rapidly expanding;

• Yannis *et al.* (2012) that focus their work on the selection of indicators on the characteristics of the metropolitan network (length and number of stations, characteristics of the city: population and density), examined the extent of metropolitan rail network versus the needs of the city, in order to assist in estimating the adequacy of a metro network. The methodology was applied to estimate the degree of adequacy of the Athens metropolitan network (Greece) in relation to the needs of the city, whose results indicated that the metropolitan Athens network could not be characterized as adequate, with future proposals being network extensions, actions which served as an initial reference point in the planning process for the future development of the Athens metropolitan system, which outlined a future metro line network of eight lines, 220 km and 200 stations, setting long-term goals for the city's transport infrastructure;

• Yin *et al.* (2018) that developed a discrete three-tier choice behavior model to analyze the dynamic demand for passenger flow under metro station disruption; and an integrated algorithm designed to manage and control the station's outage crisis by providing additional bus bridging services in order to minimize the total travel time of affected passengers and the cost of bus bridges. In addition, multimodal transport modes, including metro, bus bridge, shared bike and taxi are considered alternative options of passengers in the face of the disruption of the station. A numerical study based on the Beijing Metro network shows that additional bus bridge services can significantly eliminate the negative impact of station disruption;

• Xiong *et al.* (2013) that developed a community bus routing project for metro stations, with optimization variables that include route and progress, presenting a solution to the ideal routing project problem, with the goal of minimizing the total cost, including costs of users and suppliers, considering the demand of passenger traffic and the budgetary restrictions.

This article shows two examples of a metro-bus system developed using the Mathematica software language (Wolfram, 1996) considering a metro conjugated system with three different bus lines connected. All the data uses are hypothetical and the model can be improved to apply to a more complex transport scheme including others metro connecting options if desired (water boat transport, bicycle, etc), walking distance from metro and bus station, extra bus lines in case of accidents occurring in one of the stations, etc.

METHODOLOGY

The program for obtaining an integrated metro-bus system was developed based on the symbolic manipulation software known as Wolfram Matemathica originally developed by Stephen Wolfram. The software is continuously updated by a company called Wolfram Research that implements improvements in its computational algebra system and contains several programming libraries ready to be used for various purposes in the area of exact sciences, as is the case of engineering. Mathematica consists of two parts, which are an interface (also known as the "front end" and the kernel.) The kernel is the part that interprets the expressions and code of Mathematica and returns the results, while the graphical interface is where users work with more aesthetic graphic options and possible editions.

In order to develop an integrated metro bus system in one of the metro stops many information are necessary during the peak use such as (Correia, 2012):

- The number of passengers that uses the metro and that get off the metro on a determined platform;
- Bus distance routes to the end terminals;
- Average velocities of each bus on each of the possible routes;
- Percentages of passengers that uses the integrated bus three lines system (in this example) coming from the metro;
- Cadence of the metro service (interval time of each metro arrival at the station);
- Minimum time of permanence of each bus in the metro bus terminal;
- Minimum remain time for buses to stay in each suburban endpoints;
- Average velocity of each bus taking into consideration the stopping times during routes;
- Passengers capacity of each bus including number of seats and standing people.

Based on those data it is possible to estimate the number of buses during the peak period, the timetable for each route and the number of buses that will park at the same time at the metro station stop in order to plan enough space for them.

Figure 1 below shows a typical integrated metro system (MS) in a city connecting three different bus lines to three end terminals (TA, TB and TC).

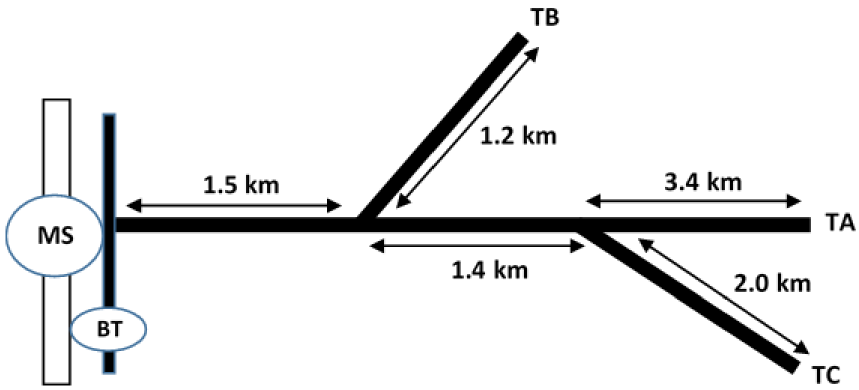

Figure 1. Transport System (not to scale).
Source: Authors example.

The total bus distance traveled DA, DB and DC for each route is equal to the sum of all the distances shown on Figure 1 for each of the bus lines.

The number of metro wagons MW per hour arriving at station is giving by equation 1 below:

$$MW = 60/Ts \qquad \text{(Equation 1)}$$

Where: Ts is the minimum time of permanence of each bus in the metro bus terminal.

The number of passengers per hour at the station NPS is calculated multiplying the number of passengers N that uses the metro and that get off the metro on the platform on peak time by the number of metro wagons per hour MW arriving at station as follows:

$$NPS = N * MW \qquad \text{(Equation 2)}$$

The number of passengers that uses each line is calculated (B_A, B_B and B_C) based on the equation below:

$$B_n = NPS * Pmb * P_n \qquad \text{(Equation 3)}$$

Where:

- Index n is equal to A for route A, B for route B and C for route C;
- Pmb is the percentage of passengers that uses the integrated bus three lines system (in this example) coming from the metro and P_n the respective percentage of passengers that comes from metro and uses lines n (A, B and C) respectively.

The number of circulating bus per hour for each line CBn can be calculated as follows:

$$CB_n = B_n/BC \qquad \text{(Equation 4)}$$

Where:

- BC is the capacity of each bus including passengers seated and standing;
- Index n is equal to A for route A, B for route B and C for route C.

On the other hand, the interval between bus departures IBD_n for each line n, in minutes, is calculated as follows:

$$IBD_n = 60/CB_n \qquad \text{(Equation 5)}$$

Where:

- CB_n is calculated based on equation 4:

The time spent from each bus line n, TB_n, in minutes, considering comings and goings can be calculated as follows:

$$TB_n = 2 * (B_n */V_n) + ts + tf \qquad \text{(Equation 6)}$$

- D_n giving in km;
- V_n giving in km/min.

Finally, the number of necessary buses NNB_n for each line n can be calculated as follows using equations 5 and 6:

$$NNB_n = TB_n / IBD_n \qquad \text{(Equation 7)}$$

Two examples of integrated metro system will be shown on the next item together with the set of data used on both cases.

RESULTADOS E DISCUSSÃO

Transport System Data

Table 1 shows the obtained hypothetical necessary data for the calculation of the integrated system for the first case.

Table 1. System set of data for the first case.

System characteristic	Values
The number of passengers that uses the metro and that get off the metro on the platform on peak time	**N=1200** passengers
Percentages of passengers that uses the integrated bus three lines system (in this example) coming from the metro	Pmb=63%
Percentage of passengers that uses bus line TA	PA=27%
Percentage of passengers that uses bus line TA	PB=53%
Percentage of passengers that uses bus line TA	PC=20%
Cadence of the metro service (interval time of each metro arrival at the station)	Cs=12 minutes
Minimum time of permanence of each bus in the metro bus terminal	Ts=5 minutes
Minimum remain time for buses to stay in each suburban endpoints TA, TB and TC	Tf=2 minutes
Average velocity of each bus including stopping times during route TA, TB and TC;	**V_A=V_B=V_C=25 km/h for all three**
Capacity of each bus including passengers seated and standing	BC=100

Source: Authors.

Table 2 shows the obtained hypothetical necessary data for the calculation of the integrated system for the second data case changing only the numbers of passengers that uses the metro on peak time and increasing the average velocities of the buses for the three lines.

Table 2. System set of data for the second case.

System characteristic	Values
The number of passengers that uses the metro and that get off the metro on the platform MS on peak time	**N=1000**
Percentages of passengers that uses the integrated bus three lines system (in this example) coming from the metro	Pmb=63%
Percentage of passengers that uses bus line A	PA=27%

Percentage of passengers that uses bus line B	PB=53%
Percentage of passengers that uses bus line C	PC=20%
Cadence of the metro service (interval time of each metro arrival at the station)	Cs=12 minutes
Minimum time of permanence of each bus in the metro bus terminal	Ts=5 minutes
Minimum remain time for buses to stay in each suburban endpoints TA, TB and TC	Tf=2 minutes
Average velocity of each bus including stopping times during route TA, TB and TC;	**V_A=V_B=V_C=30 km/h for all three**
Capacity of each bus including passengers seated and standing	BC=100

Source: Authors.

All the equations were introduced in a program developed on the "Mathematica "symbolic platform" and table 3 below shows the results for the first proposed case using the data set of table 1.

Table 3. System results for the first set of data from table 1.

System characteristic	Values
Total distance MS-TA	DA = 6.3 km
Total distance MS-TB	DB = 4.1 km
Total distance MS-TC	DC = 3.5 km
Average velocity on all routes	**V_A = V_B = V_C = 0.416667 km/min**
Number of metro wagons per hour arriving at station	5
Number of passengers per hour at the station	6000 passengers/h
Number of passengers that takes bus line B_A	1021 passengers/hour
Number of passengers that takes bus line B_B	2004 passengers/hour
Number of passengers that takes bus line B_C	756 passengers/hour
Number of circulating bus per hour for line CB_A	ROUNDED = 11 circulation buses/hour (not the number necessary per line)
Number of circulating bus per hour for line CB_B	ROUNDED = 21 circulation buses/hour (not the number necessary per line)
Number of circulating bus per hour for line CB_C	ROUNDED=8 circulation buses/hour (not the number necessary per line)
Interval between bus departures line TA	ROUNDED =5.5 minutes
Interval between bus departures line TB	ROUNDED =3.0 minutes
Interval between bus departures line TC	ROUNDED= 7.5 minutes
Time spend from bus line TA (considering comings and goings)	37.24 min
Time spend from bus line TB (considering comings and goings)	26.68 min
Time spend from bus line TC (considering comings and goings)	23.8 min
Number of necessary buses for line A	ROUNDED=7
Number of necessary buses for line B	ROUNDED = 9
Number of necessary buses for line C	ROUNDED=4
Total number of buses	20

Source: Authors.

Table 4 shows the timetable for the three bus lines based on the data obtained above. The program also gives the results in the form of graphics but due to the lot of lines under study the data on the beginning would be overlapping making it hard for readers to understand.

Table 4. Timetable results for the first studied case based on data from table 1 and 3 for the 3 bus lines.

Bus Line TA time table		Bus Line TB time table		Bus Line TC time table	
Arrivals	Departures	Arrivals	Departures	Arrivals	Departures
0 min	5 min	0 min	5 min	0 min	5 min
5.5 min	10.5 min	3 min	8 min	7.5 min	12.5 min
11. min	16. min	6 min	11 min	15. min	20. min
16.5 min	21.5 min	9 min	14 min	22.5 min	27.5 min
22. min	27. min	12 min	17 min	30. min	35. min
27.5 min	32.5 min	15 min	20 min	37.5 min	42.5 min
33. min	38. min	18 min	23 min	45 min	50. min
38.5 min	43.5 min	21 min	26 min	52.5 min	57.5 min
44. min	49. min	24 min	29 min		
49.5 min	54.5 min	27 min	32 min		
55.0 min	60 min	30 min	35 min		
		33 min	38 min		
		36 min	41 min		
		39 min	44 min		
		42 min	47 min		
		45 min	50 min		
		48 min	53 min		
		51 min	56 min		
		54 min	59 min		
		57 min	62 min		

Source: Authors.

Table 5 shows the main results obtained using the table 2 data for case 2.

Table 5. System results for the second set of data from table 2.

System characteristic	Values
Total distance MS-TA	6.3 km
Total distance MS-TB	4.1 km
Total distance MS-TC	3.5 km
Average velocity on all routes	0.5 km/min
Number of metro wagons per hour arriving at station	5
Number of passengers per hour at the station	5000 passengers/h
Number of passengers that takes bus line TA	851 passengers/hour
Number of passengers that takes bus line TB	1670 passengers/hour
Number of passengers that takes bus line TC	630 passengers/hour
Number of circulating bus per hour for line TA	9 circulation buses/hour (not the number necessary per line)
Number of circulating bus per hour for line TB	17 circulation buses/hour (not the number necessary per line)
Number of circulating bus per hour for line TC	7 circulation buses/hora (not the number necessary per line)
Interval between bus departures line TA	7 minutes
Interval between bus departures line TB	4 minutes
Interval between bus departures line TC	9 minutes
Time spend from bus line TA (considering comings and goings	32.2 min
Time spend from bus line TB (considering comings and goings	23.4 min
Time spend from bus line TC (considering comings and goings	21 min
Number of necessary buses for line A	5
Number of necessary buses for line B	6

Number of necessary buses for line C	3
Total number of buses	14

Source: Authors.

Table 6 shows the timetable obtained for the three bus lines based on the set of data from case 2.

Table 6. Timetable results for the second studied case based on data from table 2 and 5 for the 3 bus lines.

Bus Line TA time table		Bus Line TB time table		Bus Line TC time table	
Arrivals	Departures	Arrivals	Departures	Arrivals	Departures
0 min	5 min	0 min	5 min	0 min	5 min
7 min	12 min	4 min	9 min	9 min	14 min
14 min	19 min	8 min	13 min	18 min	23 min
21 min	26 min	12 min	17 min	27 min	32 min
28 min	33 min	16 min	21 min	36 min	41 min
35 min	40 min	20 min	25 min	45 min	50 min
42 min	47 min	24 min	29 min	54 min	59 min
49 min	54 min	28 min	33 min		
		32 min	37 min		
		36 min	41 min		
		40 min	45 min		
		44 min	49 min		
		48 min	53 min		
		52 min	57 min		
		56 min			

Source: Authors.

It can be seen from table 4 and 6 that the timetable changes due to the number of the metro passengers as well as with the average buses velocities. diminishing the number of metro passengers and increasing the buses velocities as on the second example less buses are necessary for each line.

The program allows correlating any change in one of the parameters involved in the model with the results of the system and consequently determining the most important data where a better research is needed.

All the data listed from table 1 can be easily changed in the program to find the new number of necessary buses for each path (TA, TB and TC) as well as the news timetables for each line. Also new bus lines can be added to the program if desired.

CONCLUSIONS

The article presented two examples of combined metro-bus system developed in the Mathematica platform and shows how to easily calculate de necessary number of buses lines for each pathway as well as the correspondent bus line timetables depending on the many parameters involved on the model system shown on the equations used.

The article also pointed out the most important parameters when you are establishing an integrated metro-bus system and showed how the number of passengers and the average bus velocity influenced on the results.

The results show that an integrated system using a symbolic language computer program, as MATHEMATICA, can contribute to urban logistics, reducing the waiting time and the time of travel of the systems. The study is an important tool that can help in the management and planning of transport systems, favoring the improvement of urban mobility, and can be applied in areas that have the same similarities, with greater or lesser demand for public transportation.

The authors would like to highlight the innovative and practical character of the use of symbolic programming that, although not shown in the article, because was not the main objective, allows for example, the inclusion of figures, maps showing the buses paths, etc, under study, which facilitates the use and understanding of the program, while running it, transforming it in a friendly use system if desired.

Lastly, we can highlight that the program can be modified to include other types of mode transport with subway connections, such as light transport on rails, ferries, by knowing the capacity of each of these transport modes in terms of passengers, demands, average speed, time per stop, if any, and other important parameters as shown on this article.

ACKNOWLEDGMENTS

We thank the reviewers for the considerations for improving the article. To the Civil Engineering Program of PUC-RJ and to the Civil Engineering Program (PEC/COPPE/UFRJ), for the support to the development of the research.

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