Abstract. This article brings a comprehensive tutorial on radio-frequency (RF) positioning in Bluetooth and Ultra-Wideband (UWB) Wireless Personal Area Networks (WPANs), carrying out an extensive literature review, providing the reader an overview of Bluetooth and UWB RF positioning state-of-the-art. It also delves into some key issues, such as multilateration (MLAT) based solutions using received signal strength and time-of-arrival, multi-slope linear regression, RF fingerprinting and the use of clustering techniques to improve its localization accuracy, the interrelation between Bluetooth versions improvements and new positioning capabilities, as well as localization prospects in future UWB networks.

1. Introduction

Bluetooth has been around for more than 20 years now (Stallings, 2005). Initially devised to replace with wireless links the serial cables connecting peripherals to computers or mobile phones, Bluetooth has evolved to support low-power short-range communications between a variety of mobile devices, from laptops and smartphones to earphones and body sensors. Its latest version (Bluetooth 4.0), with its focus on very low power consumption (thereby greatly improving battery lifetime) and reduced connection delay, made Bluetooth technology particularly well suited for indoor positioning in WPANs.

UWB is an emerging technology that offers unique advantages, as a large bandwidth and a very high time-domain resolution. The latter feature allows the application of parameters such as the channel impulse response (CIR) to locate the mobile station (MS). The use of CIR, at least theoretically, turns multi-path propagation and non-line-of-sight (NLOS) conditions - both serious impairments to RF positioning in other networks - into an additional information for accurate RF localization. UWB transmissions, due to strict power limits (FCC, 2002), have limited range, being an option for high accuracy WPAN positioning (Lau, 2011).

This article is organized as follows: first, the basics of the Bluetooth technology are presented in Section 2; Section 3 brings a literature review of positioning solutions proposed for Bluetooth networks; Section 4 introduces the fundamentals of UWB technology. Finally, Section 5 presents a literature review of localization techniques in UWB networks.

2. Bluetooth Networks

Bluetooth began as a low-power short-range wireless technology to replace cables between desktop computers and its peripherals, such as keyboard and mouse.
Therefore, at its original conception back in 1994\(^1\), it was designed to replace serial RS-232 cables. Since then, it has greatly evolved, being incorporated into a vast number of devices, such as laptops, smartphones, watches, among others. The Bluetooth specifications are written and maintained by the Bluetooth Special Interest Group (SIG), and include radio protocols and profiles\(^2\). Bluetooth is an open and royalty-free standard, which is one of the reasons why it is the de facto standard for communication in WPANs.

A Bluetooth WPAN is called a piconet, and comprises a master device and up to seven active slave devices. A piconet is a star logical topology, so all transmission must be relayed by the master - slave devices cannot communicate directly, except during the discovery phase, when a Bluetooth device sends inquiries to find nearby Bluetooth devices. Bluetooth transmissions use the 2.402 - 2.480 GHz band, which is within the Industrial, Scientific and Medical (ISM) 2.4 GHz unlicensed band. To reduce co-channel interference and interference from external sources (such as WiFi, which also uses the 2.4 GHz ISM band), Bluetooth uses Frequency Hopping Spread Spectrum (FHSS) with 79 1-MHz channels and Time Division Multiplexing (TDM) with 625μsec time slots. The master defines the piconet synchronization clock (using its own internal clock) and the frequency hopping sequence (FHS) that is going to be used by all devices in the piconet, as well as the channel allocation for the slave devices. Transmissions hop from one frequency to another every time slot. Frames might span up to 5 time slots. The Bluetooth core version 1.0 uses Gaussian Frequency-Shift Keying Modulation (GFSK), with data rates of up to 721 kbps.

### 2.1. Bluetooth Power Classes

Bluetooth devices can be classified into three power classes, as listed in Table 1. The power levels are specified at the antenna connector of the Bluetooth device. If the device does not have a connector, a reference antenna with 0 dBi gain is assumed. Class 1 devices shall implement power control, with step sizes ranging from 2 to 8 dB, being able to control their output power from 100 mW down to 2.5 mW. Between 2.5 mW and 1 mW, output power control is not used (Bluetooth Special Interest Group, 2015).

<table>
<thead>
<tr>
<th>Power Class</th>
<th>Max. Output Power</th>
<th>Min. Output Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100 mW</td>
<td>1 mW</td>
</tr>
<tr>
<td>2</td>
<td>2.5 mW</td>
<td>0.25 mW</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>N/A</td>
</tr>
</tbody>
</table>

### 2.2. Bluetooth Protocol Architecture

The Bluetooth architecture has three key elements: the profiles, the host and the controller. The profiles specify communication interfaces for Bluetooth compatible devices.

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\(^1\)Jaap Haartsen, an Ericsson Engineer, in cooperation with Sven Mattisson, designed the original Bluetooth specification. This wireless technology was named after the tenth-century King Harold Blatand - i.e., Harald Blue Tooth - who ruled over Denmark and Norway after uniting the Scandinavian tribes into a single kingdom, the same way Bluetooth unites several protocols and devices into a WPAN.

\(^2\)A profile is a set of protocols components required for the correct set up of communicating applications.
devices. Some examples of Bluetooth profiles are the cordless telephone profile (CTP) -which enables cellular phones to communicate with computers, acting as cordless phones - headset, dial-up networking profile(emulates a modem over a cellular phone) and file transfer profile (with the same functions of the file transfer protocol over Internet protocol) (Labiod et al, 2007). The host is the hardware/software that supports those profiles and that interfaces with the controller. The controller implements the Link Manager Protocol (LMP) and comprises the Bluetooth baseband processor(which provides services such as error correcting, hop sequence selection and security) and radio(that carries out modulation, demodulation, power control, among other functions). The host-controller interface(HCI) is the command interface to the Bluetooth controller - also referred to as Bluetooth module - and allows access to hardware status and control registers.

Figure 1 shows the Bluetooth protocol stack. TCS (Telephone Control Protocol Specification) provides telephony services. SDP (Service Discovery Protocol) allows Bluetooth devices to find out the services supported by other Bluetooth devices. OBEX (Object Exchange) is a data communication protocol. RFCOMM emulates an RS-232 serial interface. L2CAP (Logical Link Control and Adaptation Protocol) multiplexes data received from upper layers and adapts packet sizes between layers. The HCI allows exchanges between the host and the Bluetooth module. The LMP controls and configures links to other devices. The Baseband controls the physical link over the Bluetooth radio.

![Bluetooth Protocol Stack](image)

**Figure 1 - Bluetooth Protocol Stack**

### 2.3. Bluetooth Evolution Path

Since 1999, when version 1.0 was issued, Bluetooth SIG has adopted several enhancements to the core specification(which serves as a uniform structure for devices interoperability), as listed in Table2.

Core version 1.2 introduced Adaptive Frequency Hopping(AFH), which aims at reducing interference in Bluetooth transmissions due to the shared use of the unlicensed 2.4 GHz ISM band with other technologies, such as WiFi. The key idea is to identify ``bad'' channels and remove them from the hopping list. The channel assessment - i.e., the process of identifying the channels with high levels of interference(the ``bad'' channels) - is not defined in the SIG specification, so it is up to the Bluetooth device manufactures to select which parameter(s) to use in the assessment. The most commons are RSS and Packet Error Rate (PER): channels with high RSS levels and/or high PER
are removed from the hopping list. However, according the SIG specification, the FHS must have at least 20 channels.

Table 2 - Milestones in the Bluetooth Evolution Path.

<table>
<thead>
<tr>
<th>Core Version</th>
<th>Issue Year</th>
<th>Major Improvements</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>1999</td>
<td>-</td>
</tr>
<tr>
<td>1.2</td>
<td>2003</td>
<td>Adaptive frequency hopping,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Inquiry-based RSSI</td>
</tr>
<tr>
<td>2.0</td>
<td>2004</td>
<td>2.1 Mbps peak data rates</td>
</tr>
<tr>
<td>2.1</td>
<td>2007</td>
<td>3.0 Mbps peak data rates</td>
</tr>
<tr>
<td>3.0</td>
<td>2009</td>
<td>24 Mbps peak data rates</td>
</tr>
<tr>
<td>4.0</td>
<td>2010</td>
<td>Lower Energy Consumption, Broadcasting, Lower connection latency</td>
</tr>
</tbody>
</table>

Prior to version 1.2, the obtaining of Received Signal Strength (RSS) in Bluetooth had two relevant limitations (Wang et al, 2013):

1) to obtain the RSS indicator (RSSI) of an anchor device, the target device had to connect to it (connection-based RSSI). This introduced a delay that could render triangulation-based or fingerprinting location methods unusable, particularly for moving targets. In a 2003 study, the delay to get the RSS of a single node was 10.5 seconds - 5.3 seconds for the device discovery plus 5.2 seconds to connect to the device (Hallberg et al, 2003). The total delay exceeded 15.4 seconds with only two nodes. The longest delay to obtain a position fix reached 31.3 seconds using five anchor nodes. After this interval, a person carrying a target Bluetooth device and walking at a regular speed of 1.2 meters/second would be 38 meters away from the location where the position request was initiated;

2) the connection-based RSSI was defined as follows: if the received power was within the Golden Receive Power Range (GRPR) - a 20 dB wide interval that is assumed as the ideal received power range - the RSSI value was set to 0 dB (Bluetooth Special Interest Group, 1999). Within this interval, the RSSI was constant; therefore, no relationship could be established between received power and distance, for ranging and localization purposes. Out of the GRPR interval, the RSSI was reported in dB above (positive values) the GRPR upper limit, or below (negative values) the GRPR lower limit. Another problem was that the GRPR lower limit was defined as a function of the receiver sensitivity\(^3\), so this parameter was device specific (Bekkelien, 2012).

Core version 1.2 addressed the two aforementioned limitations, by introducing a new HCI command called INQUIRY_WITH_RSSI, which allowed including the RSSI in inquiry responses during the discovery phase (Bluetooth Special Interest Group, 2003). Inquiry-based RSSI made it possible to obtain the RSSI without the need to connect to another device, which reduced delay and overhead. Besides that, inquiry-based RSSI, unlike connection-based RSSI, was not defined in relation to the GRPR lower and upper limits, so the RSS could be reported in dBm, allowing the use of this parameter for ranging and triangulation-based positioning.

\(^3\) In the Bluetooth core specification, the receiver sensitivity is defined as the received power level for which the Bit Error Rate (BER) is equal to 0.001.
Core version 2.0 came along with Enhanced Data Rates (EDR), which achieved peak data rates of 2.1 Mbps using Differential Quadrature Phase-Shift Keying (DQPSK) modulation. This rate is almost three times higher than the previous version maximum throughput of 721 kbps using GFSK. Core version 2.1 achieved up to 3 Mbps using 8-DPSK (Differential Phase-Shift Keying) (Labiod et al., 2007). Bluetooth controllers compliant with core version 3.0+HS (High Speed) incorporate a WiFi device, reaching up to 24 Mbps over WiFi links (Bluetooth Special Interest Group, 2009).

Core version 4.0 - also referred to as Bluetooth Low Energy (BLE) - brings important improvements to Classical Bluetooth (the previous versions), mostly in energy consumption optimization and faster connections. BLE implements features such as smart host control and adjustable message length to reduce average, peak and idle mode power consumption down to 20 times, in relation to Classical Bluetooth, allowing BLE devices with small batteries to operate for a year or more (LitePoint, 2012). The smart host control places much more intelligence on the Bluetooth controller, enabling the host to sleep longer periods, being woken up by the controller only when some action needs to be performed. This saves energy in comparison to Classical Bluetooth, where the host usually consumes more power than the controller does. Adjustable message length feature also saves energy, by encapsulating messages into longer packets - fewer packets equals less overhead, reducing power consumption by the Bluetooth radio. BLE achieves faster connections by using advertising mode, when the Bluetooth device sends broadcast messages that speed up nearby devices discovery, pairing and connection (Cinefra, 2013). BLE, unlike previous versions, is not backward compatible. For this reason, the single mode devices - that support only BLE - cannot communicate with Classical Bluetooth devices. Dual mode devices support both technologies, at the cost of lower battery lifetime.

3. Positioning in Bluetooth Networks

The ubiquity of Bluetooth devices - laptops, smartphones, among others - coupled with the low cost and long battery lifetime of Bluetooth modules (particularly with the advent of BLE), makes Bluetooth networks a promising alternative for RF positioning, especially in indoor environments. Similar to WiFi localization research, the focus in Bluetooth positioning is placed in the evaluation of solutions that rely only on the pre-existing network infrastructure, i.e., that do not require any additional hardware to operate. This enables a rapid and cheap deployment of the location system, increasing its acceptability.4

Initially, the Bluetooth signal parameters available for localization purposes are the Link Quality Indicator (LQI)5, the Transmitted Power Level (TPL)6, the connection-based RSSI and the inquiry-based RSSI. However, the best option, presenting a more predictable relationship with TX-RX distance, is the inquiry-based RSSI (available from core version 1.2 onwards), which is expressed in absolute values (dBm instead of dB) and is not subject to variations due to power control (as the

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4 The acceptability can be defined as the location system's ability to be smoothly integrated with pre-existing network infrastructure (Bandara et al., 2004).
5 The LQI is a 8-bit unsigned integer - defined as a function of the BER - that expresses the perceived link quality at the receiver. The conversion from BER to LQI is device-specific.
6 Power control might be used in Bluetooth connected state to reduce interference with other Bluetooth modules and to improve battery lifetime. As it is used only in connected state, inquiry-based RSSI is not affected by it.
connection-based RSSI and TPL). LQI is related to the BER, but the conversion from BER to LQI is device/manufacturer specific. Besides that, the LQI is subject to variations due to channel conditions, such as interference, and as a result is not a viable alternative for ranging and positioning (Hossain and Soh, 2007). Connection-based RSSI is constant (0 dB) - and thereby independent of the TX-RX distance (Hallberg et al, 2003) - when the received power lies within the GRPR, which effectively also excludes it as a feasible choice for ranging and localization. For example, assuming a path-loss slope n=3 (a typical value for 2.4 GHz RF propagation at indoor environments), a Bluetooth device would report the same connection-based RSSI at 1 meter and 5 meters from the Bluetooth beacon. However, Bandara developed a positioning system in Bluetooth 1.1 using connection-based RSSI and installing variable attenuators at the Bluetooth beacons (Bandara et al, 2004). The additional loss introduced by the attenuators would then shift the received power to a level below the GRPR, where the RSSI would be linearly related to the received power (and for that reason could be used as a distance estimator for MLAT). A single beacon with variable attenuators and four antennas - placed at the corners of a 4.5 x 5.5 m² room - was deployed, and 92% of the test samples achieved location errors of 2 meters or less. Nevertheless, the proposed system requires the installation of additional hardware, which would be costly and time-consuming in a large-scale location system - for example, in a whole building - reducing the system's acceptability.

Most features related to MS position location in WiFi networks are also applicable to Bluetooth positioning: it is more relevant in indoor environments; round-trip-time (RTT) estimates are not available (which excludes time-of-arrival based MLAT as a possible location technique); the Bluetooth beacon antennas are omnidirectional, ruling out angle-of-arrival based triangulation, unless additional hardware is deployed (i.e., directional antennas). The proposed solutions for Bluetooth localization rely mostly on RSS-based MLAT (Wang et al, 2013; Hallberg et al, 2003; Cinefra, 2013; Fernandez et al, 2007; Chen et al, 2014), fingerprinting (Bandara et al, 2004; Zhang et al, 2013; Disha and Khilary, 2013), or on a combination of both (Subhan et al, 2011; Subhan et al, 2013). In Bluetooth positioning, unlike in WiFi networks, there are many MLAT based location solutions. This happens because, due to the shorter range of Bluetooth beacons, this technique is being used mostly to locate devices within each room. In that case, with the absence of obstacles such as walls along the propagation path, it is easier to model the path-loss using empirical models, allowing the use of RSS-based MLAT with good results. Proximity (i.e., returning as the MS position estimate the coordinates of the nearest Bluetooth beacon) and centroid methods are also used (Wang et al, 2013).

3.1. RSS-based MLAT solutions

There is a vast amount of published papers on Bluetooth positioning using RSS-based

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7 Nevertheless, prior to core version 1.2, it would probably be the best option for Bluetooth positioning.
8 In fact, it seems to be exclusively used for indoor localization, which is quite reasonable, if one considers the very limited range of Bluetooth devices (with the exception of Class 1 devices), if compared to the range of WiFi access points.
9 Agrawal provided a brief Bluetooth positioning taxonomy that includes some other less commonly used techniques - such as Fuzzy Logic (Agrawal et al, 2014).
MLAT, such as (Wang et al, 2013; Hallberg et al, 2003; Cinefra, 2013; Fernandez et al, 2007; Chen et al, 2014). In all of them, the following requisites are fulfilled:

1) the target MS is within range of at least three reference nodes;
2) there is a mathematical model describing path-loss as a function of distance;
3) there is an algorithm to solve the resulting overdetermined non-linear equation system. This section is devoted to studying in detail two implementations with special features: real-time calibration (Fernandez et al, 2007) and ranging with a two-slope model (Chen et al, 2014).

3.1.1. Real-time calibration

Fernandez proposed a system using real-time calibration of the path-loss to distance mapping (Fernandez et al, 2007). This scheme is able to perceive the effect of environmental changes that affect RF propagation (such as humidity, temperature, presence of people, change of furniture location, closing and opening of doors, among others), therefore improving localization accuracy. Besides using the RSS between the target and the reference nodes (Bluetooth beacons placed at known coordinates) to achieve a positioning fix - as in any other MLAT system - the solution proposed by Fernandez also employs the RSS between the anchor nodes to obtain calibration factors - called translation (from path-loss to distance) factors (Fernandez, 2007). These factors are then applied to the path-loss between the target and anchor nodes to yield distance estimates that will be used in the MLAT position fix. Consider a set of N Bluetooth beacons placed at known locations. This yields a pairwise distances matrix \( D = [d_{i,j}]_{N \times N} \), where \( d_{i,j} \) is the Euclidean distance between anchor nodes \( i \) and \( j \). Assuming that all beacons have a known constant output power\(^{10} \) \( P_t \), the path-loss between the anchor nodes can be calculated, subtracting (in the logarithmic scale) the RSS values from \( P_t \). Then, a path-loss matrix \( S = [s_{i,j}]_{N \times N} \) is obtained, where \( s_{i,j} \) is the path-loss between nodes \( i \) and \( j \).\(^{11} \) A matrix of translation factors \( \Phi = [\phi_{i,j}] \), also referred to as dynamic mapping matrix, is given by:

\[
D = \Phi S^{-1} \tag{1}
\]

where \( \phi_{i,j} \) is the factor that translates the path-loss \( s_{i,j} \) between nodes \( i \) and \( j \) into their pairwise distance \( d_{i,j} \). As \( \Phi \) is expressed as a function of the path-loss, it reflects the variations in the propagation conditions, acting as a set of calibration factors in the MLAT process. With calibration, the estimated distances between the target node and the N Bluetooth beacons at any given moment are provided by vector

\[
\hat{d} = \Phi s \tag{2}
\]

where \( s \) is a N-element vector containing the RSS values from the N anchor nodes, measured at the target device location. The following non-linear system is then

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\(^{10}\) This is a reasonable assumption, as the proposed system uses inquire-based RSSI values, and replies to inquiries during the discovery phase are not subjected to power control.

\(^{11}\) Note that both matrices \( D \) and \( S \) are symmetric and that the elements of their main diagonals are zero.
obtained:

\[
(x - x_i)^2 + (y - y_i)^2 = \hat{d}_i^2 \\
\vdots \\
(x - x_N)^2 + (y - y_N)^2 = \hat{d}_N^2
\]

where \((x_i, y_i)\) are the coordinates of the \(i\)th beacon and \(\hat{d}_i\) is the estimated distance between the target node and the \(i\)th beacon. Due to NLOS conditions and/or inherent limitations in the RSS measurement and reporting, this quadratic equation system has no closed-form solution. Fernandez (Fernandez et al, 2007) used a gradient descent method (Allison, 2001) to find iteratively the position estimate \((\hat{x}, \hat{y})\) that minimizes the following sum:

\[
(\hat{x}, \hat{y}) = \arg \min \left\{ \sum_{i=1}^{N} \left[ (x - x_i)^2 + (y - y_i)^2 - d_i^2 \right] \right\}
\]

To evaluate the location system, the authors deployed a testbed in a 5 \times 10m\(^2\) room, with four Bluetooth 2.0 beacons. Two test positions were selected: the first was at the room center, and the second close to one of the beacons. Table 3 summarizes results at the two test locations, with and without calibration for two window lengths\(^{12}\). When no calibration is used, the target node estimated location is given by a weighted centroid method. At the first test location, the use of calibration increased accuracy by 40\% for the smaller window length. At the second test location, it achieved an accuracy improvement higher than 50\% for both window lengths.

Table 3 - Positioning errors in centimeters (Fernandez et al, 2007), with and without dynamic calibration.

<table>
<thead>
<tr>
<th>Test</th>
<th>Window Lengt (sec)</th>
<th>WithoutCalibration</th>
<th>WithCalibration</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>30</td>
<td>115.4</td>
<td>69.7</td>
</tr>
<tr>
<td>1</td>
<td>60</td>
<td>100.9</td>
<td>73.2</td>
</tr>
<tr>
<td>2</td>
<td>30</td>
<td>110.4</td>
<td>49.7</td>
</tr>
<tr>
<td>2</td>
<td>60</td>
<td>108.5</td>
<td>51.0</td>
</tr>
</tbody>
</table>

3.1.2. Ranging with a Two-Slope Model

Empirical propagation models can be used to express the received power as a function of TX-RX distance, as the one defined by

\[
P(d) = P(d_0) - 10n \log_{10} \left( \frac{d}{d_0} \right)
\]

where \(P(d)\) is the RSS (in dBm) at \(d\) meters from the transmitter, \(d_0\) is a reference

\(^{12}\) The window length or integration interval is the time along which RSS samples are collected and averaged.
distance(in meters), \( n \) is the path loss exponent or slope. Assuming \( d_0=1 \) meter, one has

\[
P(d) = \beta + \gamma \log_{10}(d)
\] (6)

where \( \beta \) - also referred to as the offset - is the RSS at the reference distance and \( \gamma=-10n \) is the path-loss slope in dB/decade. Calibration campaigns are required to fine-tune such models before they can be applied to a specific environment. During these campaigns, at each selected measurement point, the distance \( d \) and the average RSS value are registered. Then, linear regression can be used to estimate parameters \( \beta \) and \( \gamma \). However, propagation characteristics might differ in regions close to and far from the transmitter. In such cases, the use of a single offset-slope pair will result in path-loss prediction errors that will escalate with distance. This will ultimately result in larger ranging errors and consequently in lower positioning accuracy for MSs in the far region. To minimize that, a breaking point might be identified, separating the near and far regions, and linear regression might be applied to each region to estimate different model parameters. The two-slope model is then given by

\[
P(d) = \begin{cases} 
\beta_1 + \gamma_1 \log_{10}(d), & d < d_{\text{break}} \\
\beta_2 + \gamma_2 \log_{10}\left(\frac{d}{d_{\text{break}}}\right), & d \geq d_{\text{break}}
\end{cases}
\] (7)

where \((\beta_1, \gamma_1)\) and \((\beta_2, \gamma_2)\) are the offset-slope pairs in the near and far regions, respectively, and \(d_{\text{break}}\) is the break point distance(in meters) in relation to the transmitter. Multi-slope linear regression has been originally used for path-loss predictions in urban microcells (Iskander and Yun, 2002), but its use can be extended to indoor environments. It is applied by Chen (Chen et al, 2014), after the collected RSS values(during the calibration or training phase) at each point are Gaussian-filtered and then averaged. The break-point - also referred to as critical point - is empirically defined. Offset-slope pairs are obtained for each Bluetooth beacon in the test bed. In the test phase, Chen used a moving average filter to smooth abrupt variations in received RSS, before ranging (Chen et al, 2014). The ranges obtained using the aforementioned two-slope model result in a non-linear equation system. Taylor series first-order expansion is used to linearize the system. In an experiment with four Bluetooth beacons, 80% of the test samples achieved a position error lower than 1.5 meters.

By comparing Figures 2c and 2d, it is possible to confirm that the use of the optimum two-slope model - illustrated in Figure 2b - results in a significant reduction of the ranging error(diminishing its average by more than 60% in that particular example), which has a direct impact on MLAT positioning accuracy. Though such improvement might not be attainable in all scenarios, it proves that a two-slope model can provide a better RSS to distance mapping in some environments.
3.2. Fingerprinting-based solutions

Unlike in cellular and UWB networks, RF fingerprinting in Bluetooth(as in WiFi) is restricted to using only RSS values. RF fingerprinting techniques for WiFi and Wireless Sensor Networks (WSNs) indoor localization can be extended to Bluetooth networks\(^{13}\). Besides that, most papers on Bluetooth positioning use RSS based MLAT, as, due to the limited range of Bluetooth beacons and devices, MLAT is being used mostly to locate devices within each room. For this reason, there are fewer papers on Bluetooth fingerprinting, with no significant new features. For that reason, this section covers in detail two special features applied to RF fingerprinting: pattern matching using signal strength difference (Hossain et al, 2013) and search space reduction using K-means clustering.

### 3.2.1. Pattern Matching using Signal Strength Difference

Manufacturing variations among Bluetooth devices might affect the way they measure RSS values. As a result, at a given location and time, different devices might measure distinct RSS values for the same Bluetooth beacon. For this reason, if the radio map\(^{14}\) is built from field measurements using a certain Bluetooth device, and another Bluetooth device is to be localized, then the fingerprinting location accuracy using this radio map may deteriorate. This is called the cross-device effect (Chen et al, 2006). To mitigate it, instead of employing absolute RSS values in the correlation function, one might use relative RSS values, i.e., their ranking or their signal strength differences (SSDs).

While absolute RSS values of a set of Bluetooth beacons might be quite

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\(^{13}\) In fact, with BLE and its emphasis on low energy consumption and enhanced battery lifetime, Bluetooth positioning might be treated in a very similar way as in ZigBee WSNs.

\(^{14}\) The radio map, also referred to as the Correlation Database (CDB), is the set of geo-referenced RF fingerprints collected (or generated via propagation simulations) during the off-line training phase of the fingerprinting algorithm.
different when measured by distinct Bluetooth devices, their ranking or their SSDs are more likely to remain the same. The greater stability of those two techniques across devices derives from the assumption that the relationship between the input signal strength at the receiving antenna and the RSS reported by the Bluetooth device is a monotonically increasing function (Krumm et al., 2003). For better understanding, consider two Bluetooth devices, MS1 and MS2, both placed at the same location, and two Bluetooth beacons, A and B. The signals from these beacons reach both MS devices with strengths \(s_A\) and \(s_B\), respectively. The RSS values informed by MS1 are RSS1A and RSS1B. The RSS values informed by MS2 are RSS2A and RSS2B. If the relationship between the input signal strength and the RSS is a monotonically increasing function, then, if \(s_A > s_B\), RSS1A > RSS1B and RSS2A > RSS2B. While the reported values (RSS1A, RSS1B) might be different from (RSS2A, RSS2B), the ranking of A and B RSS values is the same on both MSs. Besides that, one can also assume that the SSD of the signals from beacons A and B will be the same on both devices (Hossain et al., 2013), i.e., that (RSS1A-RSS1B)=(RSS2A-RSS2B).

In ranking correlation the Spearman Rank Correlation coefficient (Mathworks, 2010) is used to minimize the cross-device effect when comparing the target fingerprint (Tfing) with the reference fingerprint (Rfings). When using SSD for that same purpose, the Tfing becomes

\[ \vec{T}_k = \begin{bmatrix} \text{RSS}_1 - \text{RSS}_k \\ \vdots \\ \text{RSS}_N - \text{RSS}_k \end{bmatrix} \tag{8} \]

where \(i \neq k (k \in \{1, \ldots, N\})^\dagger\), \(N\) is the number of Bluetooth beacons and \(\text{RSS}_i\) is the received signal strength from the \(i\)th beacon. The Rfings are also built using Equation (8).

As Example 1 shows, even though there are \(\frac{N!}{2(N-2)!}\) possible pairs in a set of \(N\) beacons, there are only \((N-1)\) independent SSD values. That is why the fingerprints dimension when using SSD is \((N-1)\), if there are \(N\) beacons. The effect of this slightly smaller RF fingerprint dimension, in comparison to when absolute RSS values are used, becomes negligible for \(N>5\) (Kaemarungsri and Krishnamurthy, 2004).

**Example 1:** Given \(\vec{S}=\begin{bmatrix} -40 & -55 & -80 & -45 \end{bmatrix}^T\) containing the absolute RSS values (in dBm) of four Bluetooth beacons measured by a Bluetooth mobile device at a single location, show that the resulting SSD RF fingerprints have only 3 independent SSD values.

**Solution:** Table 4 shows vector \(\vec{S}\) and the resulting SSD RF fingerprints. Note that vector \(\vec{T}_k\), \(k = \{1, \ldots, 4\}\) is obtained subtracting the \(i\)th beacon RSS value from the RSS values of the remaining beacons. All fingerprints have only three non-null elements. Besides that, it is also possible to observe that

\[
\begin{align*}
\vec{T}_2 &= \vec{T}_1 + (S(1) - S(2))[1 1 1 1]^T \\
\vec{T}_3 &= \vec{T}_1 + (S(1) - S(3))[1 1 1 1]^T \\
\vec{T}_4 &= \vec{T}_1 + (S(1) - S(4))[1 1 1 1]^T
\end{align*}
\tag{9}
\]

\(^\dagger\) Parameter \(k\) identifies the beacon which is fixed, and whose RSS is subtracted from the RSS of the remaining beacons.
where $S(k)$ is the $k$th element of $\vec{S}$. This means that all other fingerprints can be obtained as a linear combination of the first one. Therefore, there are only 3 independent SSD values. Generalizing, if there are $N$ beacons, then only $(N-1)$ independent SSD values can be obtained.

Table 4 - Vector of absolute RSS values (in dBm) and the resulting RF fingerprints using SSD (in dB).

<table>
<thead>
<tr>
<th>$\vec{S}$</th>
<th>$T_1$</th>
<th>$T_2$</th>
<th>$T_3$</th>
<th>$T_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-40</td>
<td>0</td>
<td>15</td>
<td>40</td>
<td>5</td>
</tr>
<tr>
<td>-55</td>
<td>0</td>
<td>15</td>
<td>25</td>
<td>-10</td>
</tr>
<tr>
<td>-80</td>
<td>0</td>
<td>25</td>
<td>0</td>
<td>-35</td>
</tr>
<tr>
<td>-45</td>
<td>10</td>
<td>35</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Hossain deployed a Bluetooth testbed in a laboratory spanning an area of 214 m$^2$, with 4 fixed Bluetooth beacons and 337 training locations (Hossain et al, 2013). There, they reported a 3.4-meter average location error when using K-Nearest Neighbors (KNN) with absolute RSS values. The average error dropped to 2.6 meters when KNN was used with SSD.

3.2.2. Search Space Reduction using K-means Clustering

Search space reduction aims at reducing the time to produce a position fix in RF fingerprinting, by limiting the Rfings stored in the radio map - or Correlation Database(CDB) - that will be compared with the Tfing. It is crucial when the radio map is too large - which is the case for location systems in metropolitan areas (Campos and Lovisolo, 2009) or even in large multi-floor buildings (Campos et al, 2014). In MS-based Bluetooth indoor fingerprinting, search space reduction might also help reduce energy consumption - both of the MSs and, most importantly, of the fixed beacons - which is in keeping with one of the key features of BLE - i.e., enhancing battery lifetime. Besides CDB filtering, another alternative for search space reduction is K-means (Marsland, 2009), which is an unsupervised clustering technique$^{16}$. With clustering, the positioning fix is carried out in two phases: 1) the Tfing is presented to the trained classifier, which identifies the cluster that the Tfing belongs to; 2) the Tfing is compared only to the Rfings belonging to the selected cluster. The selected cluster - i.e., the reduced search space - is the one whose center is the closest(in the n-dimensional RSS space) to the Tfing.

During the offline training phase, first the number of clusters $N_c$ must be defined and initial values must be assigned to the $N_c$ clusters centers. Then, the following steps are taken: 1) the distances between each Rfing and each cluster center are calculated; 2) each Rfing is associated with the cluster whose center is the closest one; 3) at the end of each epoch the clusters centers are re-calculated, using the arithmetic

$^{16}$Disha and Khilary reported a median positioning error below 2 meters in a Bluetooth indoor test bed using RF fingerprinting with K-means(Disha and Khilary, 2013).
mean of the Rfings of each cluster\(^\text{17}\);  
4) the distances between each Rfing and the new clusters centers are calculated; if any Rfing switches clusters, then steps 1 to 3 are repeated; otherwise, the process ends, yielding the clusters centers vectors.  

4. UWB NETWORKS  

UWB transmissions occupy large bandwidths and have very strict output power limits. As a result, they are best suited to high-speed short-range communications. These features, coupled with their very high time-domain resolution, make UWB a promising alternative for high accuracy indoor WPAN positioning. UWB signals might be generated using very short time-domain pulses(from 10 to 1000 picoseconds), that occupy large bandwidths(from hundreds MHz to several GHz) in the frequency domain. This is called single-band UWB, also referred to as impulse radio UWB(IR-UWB). Another alternative to generate UWB signals is to divide the available UWB band into a number of non-overlapping channels\(^\text{18}\). Transmissions are made simultaneously over multiple carriers into those non-overlapping channels. This is called multi-band UWB, also referred to as multi-band orthogonal frequency division multiplexing(MB-OFDM) UWB (Lau, 2011). Each symbol is transmitted over several pulses(in IR-UWB) or over several non-overlapping channels(in MB-OFDM UWB). UWB transmissions have a low power spectral density and are spread over wide bandwidths, and do overlap with frequency bands occupied by other services. This latter characteristic arose the concerns of several segments of the telecommunications industry, afraid that the cumulative effect of UWB signals(aggregated power) could cause harmful interference to pre-existing systems, by raising the background noise\(^\text{19}\). As a result, regulatory bodies, such as the Federal Communications Commission(FCC) in the USA, issued specifications for UWB systems operation.  

4.1. Definition of UWB Technology  

The classification of a communications system into narrowband, wideband or ultra-wideband, shown in Table 5 (Lau, 2011) is based on the fractional bandwidth \(B_f\), which is given by 

\[
B_f = 2 \left[ \frac{f_H - f_L}{f_H + f_L} \right] \quad (10)
\]

where \(f_H\) and \(f_L\) are the upper and lower 10 dB points, respectively(i.e., those are the frequencies at which the output power is 10% of the maximum output power, which occurs at frequency \(f_M\), where \(f_L < f_M < f_H\). Any system with \(B_f \geq 20\%\) or that occupies a bandwidth larger than 500 MHz (the bandwidth is given by \((f_H-f_L)\)) is classified as a

\(^{17}\) At this point, another unsupervised clustering method called K-medians, instead of the arithmetic mean, uses the median. The advantage of using the median is that it is a more robust estimator, less susceptible to outliers, i.e., input vectors with much noise (Marsland, 2009). All other steps are equal for both K-medians and K-means.

\(^{18}\) MB-OFDM splits the UWB spectrum into 528-MHz wide bands. Each 528 MHz band comprises 128 carriers modulated using Quadrature Phase-Shift Keying(QPSK) on Orthogonal Frequency Division Multiplexing(OFDM) tones (Matin, 2010).

\(^{19}\) UWB signals are noise-like, due to their very large bandwidth and their very low power spectral density. For that reason, depending on the attitude of regulatory bodies, license-free operation of UWB systems could be allowed (Oppermann, 2006).
UWB system (Oppermann, 2006).

Table 5 - System type classification as a function of $B_f$

<table>
<thead>
<tr>
<th>System Type</th>
<th>Fractional Bandwidth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Narrowband</td>
<td>$B_f &lt; 1%$</td>
</tr>
<tr>
<td>Wideband</td>
<td>$1% \leq B_f &lt; 20%$</td>
</tr>
<tr>
<td>Ultra-wideband</td>
<td>$B_f \geq 20%$</td>
</tr>
</tbody>
</table>

4.2. Overview of FCC UWB Regulations

The FCC issued the UWB First Report and Order in February 2002 (FCC, 2002). It authorized three classes of UWB systems:
1) imaging systems (such as through the wall imaging, ground penetrating radar and medical imaging systems);
2) vehicular radar systems;
3) communication and measurement systems (among which RF positioning is included).

These regulations define, among other parameters, the frequency bands and the emission limits for UWB systems. Initially, the band allocated for UWB is 7.5-GHz wide, between 3.1-10.6 GHz (Oppermann, 2006). At that band, the maximum output power for indoor applications is 0.5 mW, which corresponds to a power spectral density of approximately -41.3 mW/MHz. For outdoor applications in that band, such as vehicular radar, the maximum power spectral density is 10 dB lower. Table 6 summarizes the emission limits (expressed as power spectral densities in dBm/MHz) for the authorized UWB systems in different frequency bands (Breed, 2005).

Table 6 - FCC UWB emission limits (dBm/MHz)

<table>
<thead>
<tr>
<th>Application</th>
<th>Frequency Band (GHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.96-1.61</td>
</tr>
<tr>
<td>Ground Penetrating Radar</td>
<td>-65.3</td>
</tr>
<tr>
<td>Surveillance Systems</td>
<td>-53.3</td>
</tr>
<tr>
<td>Medical Imaging Systems</td>
<td>-65.3</td>
</tr>
<tr>
<td>Indoor Comm. Systems</td>
<td>-75.3</td>
</tr>
<tr>
<td>Vehicular Radar Systems</td>
<td>-75.3</td>
</tr>
</tbody>
</table>

4.3. IEEE Task and Study Groups related to UWB

The Institute of Electrical and Electronics Engineers (IEEE) created in May 1999 the IEEE 802.15 WPAN Working Group, which aims to provide standards for low-complexity and low-power consumption wireless connectivity. Within this work group, the following currently active (as of January 2015) task and study groups are related to UWB:

- IEEE 802.15 TG3d (Task Group 3d 100 Gbit/s Wireless): its objective is to increase the data transfer speeds of 802.15.3 standard for imaging and multimedia applications, reaching data rates of up to 100 Gbps;
- IEEE 802.15 TG4r (Task Group 4r Distance Measurement Techniques): its objective is to provide communications and high precision ranging/location capability;
- IEEE 802.15 SG3c (Study Group 3e Close Proximity High-Data Rate systems): its objective is to explore the use of the 60 GHz band for WPANs;
Besides IEEE 802.15, another working group of interest to UWB is IEEE 802.19 Wireless Coexistence Working Group, which develops and maintains policies addressing issues of coexistence with current standards and/or standards under development. Due to the very large bandwidth of UWB, and as UWB systems use frequency bands overlapping with other systems, several coexistence scenarios for UWB devices have been submitted to IEEE 802.19 (Politano, 2006). It is also worth mentioning that IEEE issued a specification, IEEE 802.15.4a, which expands the original IEEE 802.15.4(WPANs) to encompass UWB technologies as well. However, it has not been ratified yet.

4.4. UWB versus Bluetooth

UWB and Bluetooth can be seen as competing technologies for WPANs: both aim at low power consumption, enhanced battery lifetime and data communications over short distances. Bluetooth is already a well-established technology, with a well-defined protocol stack. UWB, on the other hand, is still an evolving technology, and its regulation is still not completely agreed upon. Besides that, several Medium Access Control(MAC) protocols have been proposed for UWB20, and there is still ongoing research on this topic. In fact, there are many open issues in UWB MAC especially in four areas: overhead reduction, multiple access, resource allocation, and quality of service(QoS) provisioning(Zin and Hope, 2010). However, on the long run, UWB is most likely to overcome Bluetooth for short-range communication applications, due to the much higher data rates it can achieve. For example, in UWB WPANs, data rates of up to 480 Mbps will allow high speed data transfer of multimedia content, an application not supported by BLE (Matin, 2010).

5. POSITIONING IN UWB NETWORKS

As in narrowband and wideband systems previously studied, localization in UWB networks can use geometric(i.e., triangulation techniques - circular and hyperbolic MLAT;multi-angulation) and RF fingerprinting. However, not all those methods can take full advantage of the UWB signals very high time-domain resolution.

5.1. Geometric positioning

5.1.1. Multi-angulation

Angle-of-Arrival(AOA) positioning, also referred to as Direction of Arrival(DOA), requires the deployment of antenna arrays at the reference nodes, what increases the localization system infra-structure cost. Besides that - and maybe more importantly - AOA positioning accuracy is severely affected by multi-path and NLOS propagation conditions. As a result, AOA is not a viable alternative for practical deployments of UWB indoor localization systems (Yang, 2011).

5.1.2. TOA-based MLAT

Time-of-Arrival(TOA)-based MLAT, as in narrowband and wideband systems, is negatively affected by multi-path and NLOS conditions. However, high time-domain

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20 Zin and Hope provided a summary of proposed UWB MAC protocols (Zin and Hope, 2010).
resolution of UWB signals enable an accurate TOA estimation\(^{21}\). Aside from detecting the TOA as exactly as possible, it is also important to identify the channel conditions to improve circular MLAT accuracy. Three basic channel conditions have been categorized (Pahlavan et al., 1998):

1) Dominant Direct Path (DDP), when the line-of-sight (LOS) component (the first received multi-path component) is the strongest;
2) Non-Dominant Direct Path (NDDP), when there is a LOS component, but it is not the strongest;
3) Undetected Direct Path (UDP), when there is no LOS component, i.e., the direct path between the transmitter and the receiver is obstructed.

First, the channel condition must be identified, primarily to find out if there is a direct path (LOS). This first step is called NLOS detection. Second, if NLOS conditions have been verified in the first step, NLOS mitigation techniques should be applied. Schroeder presents several methods for NLOS detection (Schroeder, 2007). Lie and See analyze some alternatives for NLOS mitigation (Lie and See, 2011). All these measures help improving ranging (obtained from the TOA measurements), and, as a result, the positioning accuracy as well. The high time-domain resolution of UWB signals enhances the NLOS detection and mitigation capabilities of those algorithms.

5.1.3. RSS-based MLAT

RSS-based MLAT, even though a viable alternative for UWB localization, does not benefit from the UWB high time-domain resolution, resulting in positioning accuracies similar to those achieved by narrowband or wideband systems. For example, Amiot and Laaraiedh deployed four UWB receivers at fixed locations in a 350 m\(^2\) floor in an office building (Amiot and Laaraiedh, 2013). A transmitter was moved along 302 positions selected within that floor, generating IR-UWB signals in the 3-7 GHz band, with an output power of 26 dBm. The RSS at each receiver, for each transmitting position, was registered, together with the TX-RX distance. Those values were used to calculate log-normal shadowing path-loss models for each receiver location, employing linear regression. The final position estimate was obtained in a two-step process: first, the room where the MS was located was identified using RSS-based fingerprinting; second, the MS location within the selected room was estimated using RSS-based MLAT. In the first step, a 98.7\% room identification accuracy was reported. In the second step, a median positioning error of approximately 3 meters was achieved.

5.2. RF Fingerprinting

CIR-fingerprinting takes full advantage of the huge UWB bandwidth and yields high position accuracy. This method takes factors which impair positioning accuracy in narrowband and wideband systems - multi-path propagation and NLOS conditions - and turns it into an advantage to RF localization\(^{22}\).

The CIR to the impulse transmitted by the \(i\)th anchor node, measured at the \(j\)th measurement point (i.e., at the \(j\)th reference point of the radio map), is given by

\(^{21}\)Yang analyzes several algorithms for TOA estimation in UWB networks (Yang, 2011).
\(^{22}\)Some results show that CIR-fingerprinting location accuracy is higher in NLOS than in LOS conditions (Wasim and Ben, 2007).
\[ h_{i,j}(t) = \sum_{k=1}^{M} a_{i,j,k} \delta(t - \tau_{i,j,k}) e^{-j\theta_{i,j,k}} \] (11)

where \( a_{i,j,k} \), \( \theta_{i,j,k} \) and \( \tau_{i,j,k} \) are the amplitude, phase and delay of the \( k \)th received multi-path component from the \( i \)th anchor node transmission, measured at the \( j \)th reference point\(^23\); \( M \) is the maximum number of resolved multi-path components and \( \delta(t) \) denotes the impulse function. If at reference point \( j \), less than \( M \) multi-path components are detected from the impulse transmitted by anchor node \( i \), then zero-padding is used to complete vector \( h_{i,j} \), which will then have \( M \) elements, for any \( i=1,\ldots,N \) and \( j=1,\ldots,N_R \). Note that \( N \) is the number of anchor nodes and \( N_R \) is the number of entries in the radio map.

Equation (11) shows that, in multi-path propagation conditions, the receiver detects multiple delayed and attenuated copies of the transmitted impulse. By taking the CIR with respect to different anchor nodes at the same position \( j \), it is possible to build an RF fingerprint given by

\[ R_j = [h_{1,j}(t) \ldots h_{N,j}(t)] \] (12)

Each element of the fingerprint is a \( M \)-element vector, so the CIR-based fingerprint is a \( M \times N \) matrix. The CIR is expected to carry the multi-path information that is unique to each location.

Assume that the target device measures the CIR to the \( N \) anchor nodes, yielding the target RF fingerprint given by

\[ T = [h_1(t) \ldots h_j(t)] \] (13)

where \( h_i \) is the CIR with respect to the \( i \)th anchor node. The target node position estimate is provided by the coordinates of reference point \( j \), whose CIRs (stored in the reference fingerprint \( R_j \)) maximize the sum of the CIR cross-correlation coefficients, i.e

\[ j = \text{argmax} \sum_{i=1}^{N} |C_{i,j}| \] (14)

where \( j \in W, W=\{1,\ldots,N_R\} \)\(^24\), \( N_R \) is the number of georeferenced points in the radio map and \( C_{i,j} \) is the CIR cross-correlation coefficient between the target (\( T(i) = h_i \)) and reference (\( R_j(i) = h_{i,j} \)) CIRs with respect to the \( i \)th anchor node, given by

\[ C_{i,j} = \frac{E\{m n^*\} - E\{m\} E\{n^*\}}{\sqrt{(E\{|m|^2\} - |E\{m\}|^2)(E\{|n|^2\} - |E\{n\}|^2)}} \] (15)

where \( m=h_{i,j} \) and \( n=h_i \); \( n^* \) denotes the conjugate complex of \( n \) and \( E\{\cdot\} \) denotes the expectation operator. To improve positioning accuracy, a form of KNN is used: instead of returning only the coordinates of the reference point identified by index \( j \), that maximizes Equation (14), all reference points where the sum of the CIR

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\(^23\) Each reference point, stored in the radio map or CDB, is georeferenced: there is a pair of coordinates (for planar positioning) associated with it.

\(^24\) Note that \( W \) is simply the set of reference point indexes in the radio map.
cross-correlation coefficients is greater than a threshold $C_{th}$ are selected. The target node position estimate is then given by the arithmetic mean of the coordinates of the selected reference points, or the position fix might return an ambiguity region, formed by the coordinates of the selected reference points.

Wasim and Ben carried out indoor measurements in the FCC allocated UWB band (3.1-10.6 GHz) (Wasim and Ben, 2007). Some locations have been selected for the UWB impulse radio. Then, a set of measurement points was identified in the indoor testbed. For each transmitter location, at each measurement position, 1601 discrete frequency samples were taken over the 7.5-GHz wide band, and the Channel Transfer Function (CTF) was calculated (Sindi, 2013). The CIR was obtained calculating the Inverse Discrete Fourier Transform (IDFT) of the CTF. On average, the ambiguity regions returned by the position fixes had radiuses of approximately 2 cm. Nevertheless, such a high accuracy does not come without a price, as the training phase of CIR-fingerprinting is time consuming, complex (as it requires specific equipment, such as vector network analyzers) and the resulting radio map might be very large. For these reasons - that are of paramount importance, if one considers a practical implementation of such a positioning system - this method is more suited only to small-size indoor environments.

6. SUMMARY
This paper presented the fundamentals of Bluetooth and UWB positioning. A brief review of the Bluetooth core versions, along its evolution path, has been provided, highlighting the improvements in the Bluetooth specification that were relevant to RF positioning, such as inquiry-based RSSI. Two techniques to enhance RSS-based MLAT in Bluetooth networks have been studied: real-time calibration and ranging with two-slope models. Following, the basics of the UWB technology have been introduced: the types of UWB signals (IR-UWB and MB-OFDM UWB), the allocated frequency bands, the output power limitations, among other relevant information. Finally, the applicability in UWB networks of geometric and scene analysis localization techniques have been analyzed, and CIR-fingerprinting, which achieves average location errors around 2 centimeters, has been presented.

REFERENCES

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25 As $|C_{i,j}| \leq 1$, for any $i=1,...,N$ and $j=1,...,N_R$, with $N$ anchor nodes, we would have $0 \leq C_{i,j} \leq N$. A typical condition would be $C_{th} = 0.9N$. However, the value of $C_{th}$ must be carefully chosen, according to the specific propagation conditions at the place where the positioning system is to be deployed. Its value could then be determined using the fingerprints collected during the training phase.
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