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AN IF-ITEM-DELETED SENSITIVE ANALYSIS OF CRONBACH'S ALPHA TECHNIQUE USING SIMULATED ANNELING ALGORITHM

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Abstract

This work proposes a nonlinear model and also a solution method to improve the overall Cronbach's alpha coefficient technique, *by grouping items of an instrument (questionnaire) that should be considered to deletion. The classical method called if-item-deleted Alpha also improves the overall reliability, however it considerers the deletion of only one item per time. Due to the combinatorial nature of the proposed model, a simulated annealing algorithm was implemented in order to achieve optimal or near optimal solutions. We presented an example for a simulated questionnaire with 15 items and 22 respondents. The obtained solution indicates the deletion of 8 items for maximum alpha or 2 items to exclude the most problematic items.*

Key-words*: Cronbach's Alpha; Non Linear Programming; Questionnaire; Simulated Annealing*

1. Introduction

Validity and reliability are two fundamental concepts in the evaluation of instruments (questionnaires) that can measure concepts, psychomotor skills or affective values. Validity is concerned with the extent to which an instrument is intended to measure and reliability is concerned with the ability of measuring with consistency. It should be noted that the reliability of an instrument is closely associated with its validity. An instrument cannot be valid unless it is reliable. However, the reliability of an instrument does not depend on its validity (TAVAKOL; DENNICK, 2011).

One tool that helps researches to assure if their instruments are reliable is the Cronbach's alpha coefficient and the if-item-deleted alpha sensitive analysis. The first one analyses the instrument as a whole and the second analyses each instruments' item, one per time, and informs the overall increase or decrease of alpha in case of item deletion (HORA *et al*., 2010).

We propose a nonlinear model that makes the same sensitive, however our model considers the deletion of one or more items at the same time trying to find better results than the classical method. As this problem has a combinatorial nature, in order to find an optimal or near optimal solution we have applied a simulated annealing algorithm.

Simulated annealing is a successful stochastic global optimization metaheuristic that was independently developed by Kirkpatrick *et al.* (1983) and by Cerny (1985). The authors based their study on the Metropolis-Hastings algorithm created by Metropolis *et al*. (1953). According to Brownlee (2011), the strategy of a successful technique, that finds global solutions and avoids being trapped in local minima, is to search the problem space and reject or accept solutions based on a probabilistic function that accepts a variety of solutions in the beginning of the algorithm and then becomes more discerning as the number of iterations increase, as simulated annealing does.

This paper is arranged as follows. In Section 1, the main purpose of our model is explained. In Section 2, details about Cronbach's alpha and if-item-deleted alpha are given. In Section 3, the nonlinear model is presented In Section 4, the application of simulated annealing algorithm is explained. In Section 5, numerical examples are provided, and, finally, in Section 6, this paper is concluded with an analysis of the prosed model and algorithm.

2. Cronbach's Alpha and If-Item-Deleted Alpha

Cronbach's alpha is a popular method in quantifying the reliability of the information of several items in surveys (AELST, 2006). Alpha was developed by Lee Cronbach in 1951 to provide a measure of reliability of a test or scale, expressed as a number between 0 and 1, although negative values can be obtained (GLIEM; GLIEM, 2003).

Tavok and Dennick (2011) affirm that alpha is an important concept in the evaluation of assessments and surveys. It is mandatory that assessors and researchers should estimate this quantity to add validity and accuracy to the interpretation of their data. The Cronbach's alpha is defined as follows:

$$\alpha = \frac{K}{(K-1)} . \left(1 - \frac{SK}{SR}\right)$$

where

K = represents the number of items considered;

SK = represents the total sum of the variance for each item;

SR = represents the total sum of variance for all cases (for each interviewed stakeholder).

A high value of alpha near 1, is caused by high variance (explained variance), meaning that there is a wide spread of scores in a questionnaire, and also caused by low error variance (unexplained variance) (MONTEIRO; HORA, 2014).

George and Mallery (2003) provide the following rule to interpret the alpha results: Excellent (values greater or equal to 0.9); Good (values greater or equal to 0.8); Acceptable (values greater or equal to 0.7); Questionable (values greater or equal to 0.6); Poor (values greater or equal to 0.5) and Unacceptable (values lesser to 0.5).

For the if-item-deleted alpha sensitive analysis, which shows how alpha would change if a question was not on the test, lower result means that the question should not be considered to deletion, because it would lower the overall alpha. Questions with higher results tend to have low inter-item correlations and should be considered to deletion because they improve the overall alpha (STREINER, 2003).

The derived metric if-item-deleted alpha indicates which questions are not conforming to the overall conceptual framework of the instrument (ALLEN *et al.*, 2008).

3. Nonlinear Model

The proposed nonlinear model was constructed to analyze the deletion of groups of items that the classical model fails to realize, since it considers the deletion of one item per time. The nonlinear model selects groups of deletion that achieves better or equal (in the worst case) improvements in the Cronbach's alpha value. The complete model is given by:

Maximize $\frac{K}{(K-1)}.\left(1-\frac{ST}{SR}\right)$ (1)

s.t

$$K = \sum_{i=1}^{n} x_i \quad (2)$$

$$K \geq 2 \quad (3)$$

$$ST = \sum_{i=1}^{n} SK_i . x_i \quad (4)$$

$$C_j = \sum_{i=1}^{n} p_{ij} . x_i ; j = 1; \dots ; m \quad (5)$$

$$\mu = \frac{\sum_{j=1}^{m} C_j}{m} \quad (6)$$

$$SR = \frac{\sum_{j=1}^{m} \left(C_j - \mu\right)^2}{m-1} \quad (7)$$

$$x_i \in \{0; 1\} \quad (8)$$

Where:

K = linear variable that represents the total number of items considered;
x_i = binary variable that indicates if an item should be deleted ($x_i = 0$) or not ($x_i = 1$);
C_j = linear variable that represents the sum of evaluations made by a case "j" ($1; \dots ; m$) for each not deleted item;
ST = linear variable represents the total sum of the variance for each "i" item;

SK_i = variance of each "i" item;
p_{ij} = represents the evaluation of the item "i" by the case "j";
m = represents the total number of cases considered;
μ = represents the average of the sum of the evaluation made by all cases;
SR = represents the total sum of variance for all cases.

The objective function to be maximized is showed in (1) that refers to the Cronbach's alpha value. Constraint (2) imposes a total minimum of items to be maintained in the instrument, as Constraint (3) forces this number to be at least 2, both constraints are necessary in order to make the model feasible. Constraint (4) represents the total sum of the variance for each maintained item. Constraint (5) represents the sum of evaluations made by a respondent "j" for each maintained item, and Constraint (6) represents the average of the sum of the evaluation made by all respondents. Constraint (7) represents the total sum of variance for all respondents. Finally Constraint (8) represents the binary variables that indicates if an item should be deleted ($x_i = 0$) or not ($x_i = 1$).

4. Simulated Annealing

Here, we propose a simulation annealing algorithm to solve the proposed nonlinear model. The pseudocode of the algorithm is presented in Table 1.

Table 1 – Pseudo code of the proposed algorithm

1.	Initialize $t_0 \leftarrow 1$, $ArrayP_s$ (All variables with value equal 1)
2.	$ArrayP_{best} \leftarrow ArrayP_s$
3.	**Repeat** (block "a")
4.	Cooling Schedule $\leftarrow t = 0.9t_0$
5.	**Repeat** (block "b")
6.	D = 1
7.	**Repeat** (block "c")
8.	$ArrayP_r$ (Random deletion of D variables)
9.	$\Delta = \frac{Cost(ArrayP_r) - Cost(ArrayP_s)}{Cost(ArrayP_s)}$
10.	$r_a \leftarrow$ random continuous number between 0 and 1
11.	**if** $\Delta < 0$ **OR** $r_a \leq e^{\frac{-\Delta}{t}}$ (Acceptance function) **then**
12.	$ArrayP_s \leftarrow ArrayP_r$
13	**end if**
14.	**if** Cost ($ArrayP_r$) < Cost ($ArrayP_{best}$) **then**
15.	$ArrayP_{best} \leftarrow ArrayP_r$
16	**end if**
17.	$D \leftarrow D + 1$
18.	**until** D = Maximum of allowed items deletions (end of block "c")
19.	**until** $ArrayP_{best}$ stabilization or max number of iterations (end of block "b")
20	$t_0 = t$
20.	**until** frozen state or max number of iterations (end of block "a")
21.	**Return**S_{best}; $ArrayP_{best}$

In the Table 1, line 1, the initial parameters values t_0 (temperature) and ArrayPs are set. In the initial solution $ArrayP_s$ all variables are equal to 1. In line 2, the best solution is initialized as $ArrayP_s$.

The temperature t_0 must be higher enough in the beginning. In this way, worse solutions can be accepted, to avoid being trapped in a local minimum. Then, the temperature must slowly decrease until it reaches a very low value, and in this way, only solutions that improves the quality of the current solution is accepted. The value of the temperature t_0 considered depends on the values being compared of the objective function. In this type of problem, these values can vary greatly from instance to instance, so we propose comparing the percentage changes in the objective function. With this approach, we expect that the values of the objective function in the comparisons will be between -1 and 1, so an adequate higher temperature would be 1.

The first block of repetition, named "block a", begins in line 3. In this block, we choose a cooling scheduling function (line 4) that makes t_0 decrease, and this value is updated in each repetition of this block. We have chosen the geometric scheduling proposed by Kirkpatrick et al. (1953), which is t = ϕt0. The value of ϕ can vary between 0.8 and 0.99. From experiments, we found out that the value 0.9 was adequate for the problem instance. Other cooling schedules exist in the literature, some authors such as Hajek (1988) and Nourani and Andresen (1998) have studied and compared many of them. However, we did not extensively test other cooling schedules, because we found that the geometric scheduling was efficient in obtaining optimal or near optimal solutions with few iterations. "Block a" must be repeated until it reaches a frozen state (a very low temperature, where the system is steady) or the number max of iterations. In our case, a frozen state can be reached at 100 iterations.

In line 5 begins the "block b" which is nested in "block a", and must be repeat until $ArrayP_{best}$ stops improving temperature or the maximum number of iterations is reached. In this block the initial value of allowed deletions is set to one ($D = 1$) in line 6. We suggest that a number of 25 iterations should be enough to stabilize most of the solutions in each temperature level, without compromising the value of the global minimum solution.

In line 5, "block c" begins, in the "block b". Each $ArrayP_r$ (line 8) is composed only by binary values (0 or 1) representing the set of items that are maintained (value of 1) or deleted (value of 0). The number of allowed deletions is controlled by the variable D, and it is increased by 1 after each iteration. The cost function (alpha value calculated by the model) of the $ArrayP_r$ is defined and compared with the cost function of $ArrayP_s$ through the equation 9 (line 9):

$$\Delta = \frac{Cost(ArrayP_r) - Cost(ArrayP_s)}{Cost(ArrayP_s)} \qquad (9)$$

In line 10 r_a represents a random continuous number between 0 and 1. The acceptance function is represented by $e^{\frac{-\Delta}{t}}$, which is the probability of accepting a worse move, and as the temperature decreases, the probability of accepting a worse move also decreases. Therefore, we can define that:

$$r_a \leq e^{\frac{-\Delta}{t}} \qquad (10)$$

If Δ (9) is negative or r_a (10) holds true, then the values of the $ArrayP_s$ will be substituted by the values of $ArrayP_r$ (line 12), and if the cost of $ArrayP_r$ is less than the cost of $ArrayP_{best}$, then the value of the $ArrayP_r$ becomes the new $ArrayP_{best}$ (line 15), and "block c" must be repeated until D reaches its maximum value (depending on the total number of items).

5. Methodology Application

In order to demonstrate the results that can be obtained by solving the proposed model with the proposed algorithm, a questionnaire results with simulated data is used as an illustrative example (Table 2). The columns represent the items and the lines represent the respondents (HORA *et al.*, 2009; HORA *et al.*, 2010):

Table 2 – Questionnaire

	Q01	Q02	Q03	Q04	Q05	Q06	Q07	Q08	Q09	Q10	Q11	Q12	Q13	Q14	Q15
1	1	0.75	0.75	0.75	1	1	0.75	0	1	1	1	1	0	1	1
2	1	0.75	0.75	0.75	1	1	1	0	1	1	1	1	0	1	1
3	1	0.5	0.75	0	1	0.75	1	1	1	1	1	1	1	0	1
4	1	1	1	0.75	0.75	1	1	0	1	0.75	1	1	0.75	0.75	0.75
5	1	1	1	0.75	0.75	1	1	0	1	0	1	1	0.75	1	1
6	1	0.75	0.25	0.75	1	1	1	0	1	0.5	1	0.75	0.75	1	1
7	0.75	0.5	0.5	0.75	0	0.75	1	0	0	1	1	0	0	1	1
8	1	0	0.25	1	0.75	0.75	0	0	1	0.5	1	0.5	0	0.25	1
9	0.75	0.75	0.75	1	0.75	0.75	1	0.5	1	0.5	0.75	0.75	0.25	1	0.75
10	0.75	0.25	0.5	0	0.5	0.75	0	0	1	0.75	1	0.75	0.25	0.5	1
11	1	0.75	0.5	0.75	1	1	1	0	1	0.5	1	0.75	1	1	1
12	1	0.75	0.75	0	1	0.75	1	1	1	1	1	1	1	0	1
13	1	1	1	0	0.75	1	1	1	0.75	1	0	0	0	0	0
14	0.75	0.5	1	1	1	1	1	1	1	1	1	1	1	1	1
15	1	1	1	0.75	1	1	0.5	0.75	1	1	1	1	0.75	1	1
16	1	0.5	1	1	1	1	1	1	1	1	1	1	1	1	1
17	0.75	0.75	0.75	0.75	1	1	1	0	1	1	1	1	0	1	1
18	0.75	1	1	0.5	0	0.75	1	0.25	1	1	0	0	0.5	1	1
19	0.75	0.5	0.5	0.75	0	0.75	1	0	0	1	1	0	0	1	1
20	0.75	0.5	0.75	0	0	0.75	0.75	1	0.5	1	0.75	0	0	1	0.75
21	1	0.5	1	0.75	1	1	0.5	0	1	1	0.75	0.25	0.75	1	1
22	0.75	0.75	1	0.75	1	1	0.5	0.25	1	1	1	0.5	0.75	1	1

The obtained solution improves the alpha coefficient value, which is demonstrated in Table 3 ($D = 0$; indicates the full instrument without deleted items). Note that the algorithm found the maximum number of items to be deleted equal to 8:

Table 3 – Results

D	Q01	Q02	Q03	Q04	Q05	Q06	Q07	Q08	Q09	Q10	Q11	Q12	Q13	Q14	Q15	Alpha
8	1	**0**	**0**	**0**	1	1	**0**	**0**	1	**0**	1	1	1	**0**	**0**	0.822173291
7	1	1	1	**0**	1	1	**0**	**0**	1	**0**	**0**	1	1	**0**	**0**	0.812865105
6	1	1	1	1	1	1	**0**	**0**	1	**0**	**0**	1	1	**0**	**0**	0.786149604
5	1	1	1	**0**	1	1	1	1	1	**0**	**0**	1	1	**0**	**0**	0.771634028
4	1	**0**	1	1	1	1	**0**	**0**	1	**0**	1	1	1	1	1	0.774110032
3	1	1	1	1	1	1	**0**	**0**	1	**0**	1	1	1	1	1	0.77013286
2	1	1	1	1	1	1	1	**0**	1	**0**	1	1	1	1	1	0.760141376
1	1	1	1	1	1	1	1	**0**	1	1	1	1	1	1	1	0.723607498
0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0.69096949

The decision maker should decide which set of items should be deleted in order to improve the overall alpha coefficient, using as criterion the maximum alpha value (in this case deleting Q02, Q03, Q04, Q07, Q08, Q10, Q14 and Q15) or choosing any deletion level. A reasonable decision would be to choose $D = 2$ and delete items Q08 and Q10, because both items appear in almost all levels of deletion. However if the questionnaire is still under the reviewing stage the maximum alpha value criterion can be useful to reformulate the items indicated to deletion. For $D = 1$ we have the classical if-item-deleted alpha technique.

6. Conclusions

A nonlinear model was developed in order to maximize the overall coefficient of Cronbach's alpha value. Moreover, a simulated annealing algorithm to solve the proposed model was introduced. Through a numerical illustrative example, we have demonstrated improvement in the process of obtaining alpha values. Now, the decision maker can choose the ideal quantity of items to be deleted or substituted in his research.

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