Philosophical Methodology and the Mathematization of Pedagogy
Freeing Children's Imagination through Philosophy

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Abstract:
This paper traces the genealogy of a long-enduring controversy in Western philosophy viz, whether philosophic and mathematical methodologies are equal but separate and distinct approaches to rational inquiry, or whether one is superior to the other from the standpoint of epistemology, and, ultimately, a pedagogy which supports and promotes conceptual and critical thinking. With the Socratic teacher in mind, philosophic methodology, viewed by Plato as a dialectical process of free-ranging inquiry, compelled him to distinguish the work of philosophy from that of mathematics, since philosophic methodology could not function unless freed from the constraints contained in a system built on axioms, propositions, and images. In his dialogue *Meno,* Socrates' pedagogic demonstration, supported by the unique capacity of mathematics to provide epistemic closure, foreshadowed a slow-growing rift between philosophy and mathematics, one which came to a head in the 17th century rationalism of Descartes. In Descartes' work on methodology, mathematics, by overriding the seemingly inconclusive meanderings of philosophy, became the supreme pathway to knowledge. Philosophy, thereafter, had to fight for its life, while pedagogy, questing more and more for "completeness" through formalization, ironically moved toward a fragmentation and reductionism which a mathematized Cartesian methodology supported. The curriculum divisions which resulted were presaged in Descartes' writing on methodology; the isolation of the individual learner from the larger community of discourse was supported by Descartes' "cogito." Suggested here is that a pedagogic approach to the teaching of mathematics, which extends beyond the limitations of the "single solution" methodology utilized in *Meno*, is possible—an approach which allows pedagogy to spread the Socratic use of dialectical thinking even into that sphere of apodicticity. This suggests that an alternative to the monistic rationalism of Descartes' mathematized methodology is possible for pedagogy in general, now bringing back into play what Descartes found expendable—viz. dialectical philosophy itself, dialogical inquiry, history, literature, and the arts. Had Socrates, who came to philosophy primarily as a teacher fascinated with intellectual paradoxes, explored, in *Meno*, the *pedagogic* possibility of a dialectical approach to mathematics, his larger work in dialectical pedagogy might have been less teacher-centered—notwithstanding his ironic "ignorance"; and given his historic significance as the most celebrated teacher in Western culture, modern pedagogy might have been less overwhelmed by the Cartesian drive toward mechanized and reductionist "single-solution," non-communal teaching and thinking.

Key words: Cartesian methodology; mathematical method vs. dialectical method; complete education; Feyerabend.

Metodología filosófica y Matematización de la pedagogía

Resumen:
Este trabajo remonta la genealogía de una extensa controversia que abarca la filosofía occidental acerca de si las metodologías filosóficas y matemáticas son iguales pero separadas y acercamientos distintos a la investigación racional, o si una es superior a la otra del punto de vista de la epistemología, y, en última instancia, una pedagogía que apoya y promueve el pensamiento conceptual y crítico. Con el profesor socrático en mente, la metodología filosófica, vista por Platón como proceso dialéctico de investigación libre de clasificaciones, lo obligó a que distinguiera el trabajo de filosofía del de matemática, puesto que la metodología filosófica no podría funcionar a menos que estuvo liberada de los apremios contenidos en un sistema construido sobre axiomas, proposiciones, e imágenes. En su diálogo *Menón*, la demostración pedagógica de Sócrates, apoyada por la capacidad única de la matemática de proporcionar un cierre epistemológico, presagió una grieta de crecimiento lento entre la filosofía y la matemática, que llegó a su punto más alto con el racionalismo de Descartes, en el siglo XVII. En el trabajo de Descartes sobre la metodología, la matemática, eliminando los parloteos aparentemente poco concluyentes de la filosofía, se convirtió en el camino supremo al conocimiento. La filosofía, a partir de allí, tuvo que luchar para su vida, mientras que la pedagogía, buscando cada vez más "la completitud" a través de la formalización, se movió irónicamente hacia una fragmentación y un reduccionismo que una metodología cartesiana matematizada dio sustento. Las divisiones del currículo resultantes fueron presagiadas en los escritos y la metodología de Descartes; el aislamiento del aprendiz individual de la comunidad más amplia de discurso fue apoyado por el "cogito" de Descartes. Se sugiere aquí que es posible un acercamiento pedagógico a la enseñanza de la matemática, que se extiende más allá de las limitaciones de la metodología de la "solución única" utilizada en el *Menón.* Este acercamiento permite que la pedagogía extienda el uso socrático del pensamiento dialéctico incluso a la esfera de lo apodíctico. Esto sugiere que una alternativa al racionalismo monístico de la metodología matematizada de Descartes es posible para la pedagogía en general, toda vez que ella pone en juego nuevamente lo que Descartes encontró sacrificable, esto es, la propia filosofía dialéctica, la investigación dialógica, la historia, la literatura, y las artes. Si Sócrates, quien llegó a la filosofía sobre todo como un profesor fascinado por las paradojas intelectuales, hubiera explorado, en el *Menón*, la posibilidad *pedagógica* de un acercamiento dialéctico a la matemática, su trabajo más amplio en la pedagogía dialéctica pudo haber estado menos centrado en el profesor, a pesar de su "ignorancia irónica"; y dada su significación histórica como el profesor más celebrado de la cultura occidental, la pedagogía moderna se podría haber visto menos abrumada por el impulso cartesiano hacia soluciones "únicas", mecanizadas y reduccionistas, y hacia la enseñanza y el pensamiento no comunitarios.

Palabras clave: Metodología cartesiana; método matemático versus método dialéctico; educación completa; Feyerabend.

Metodologia filosófica e matematização da pedagogia

Resumo:
Este trabalho remonta a genealogia de uma longa controversa existente na filosofia ociendental a respeito das metodogias filosóficas e matemáticas, se elas são parecidas mas separadas e têm diferentes aproximações à investigação racional, ou se uma é superior a outra do ponto de vista da epistemologia, e, em última instância, uma pedagogia que apoia e promove o pensamento conceitual e crítico. Com o professor socrático em mente, a metodologia filosófica, concebida por Platão como um processo dialétido de investigação livre de classificações, o obrigou a distinguir o trabalho da filosofia da matemática, porque a metodologia filosófica não poderia funcionar a não ser que estivesse livre das coerções exercidas pelo sistema constrído sobre axiomas, proposições e imagens. Em seu diálogo *Menon*, a demonstração pedagógica de Sócrates, apoiada pela capacidade única que tem a matemática de proporcionar um fechamento epistemológico, indica um pequeno espaço de crescimento entre a filosofia e a matemática, que chegou ao seu ápice com o racionalismo de Descartes, no século XVII. No trabalho de Descartes sobre a metodologia, a matemática, eliminando as aparecentes conclusões inconclusivas da filosofia, se converteu no caminho supremo ao conhecimento. A filosofia, a partir de então, teve de lutar pela sua vida, enquanto que a pedagogia, buscando cada vez mais a "completude" através da formalização, moveu-se ironicamente à uma fragmentação e a um reducionismo que uma metodologia cartesiana matemática deu sustento. As divisões do currículo resultantes foram previstas nos escritos de Descartes sobre a metodologia; o isolamento do aprendiz em relação a sua comunidade mais ampla de discurso foi apoiado pelo "cogito" de Descartes. Sugere-se, aqui, que é possível uma aproximação pedagógica ao ensino da matemática, que se estende para além das limitações da metodologia de solução única utilizada no *Menon*. Esta aproximação permite que a pedagogia estenda o uso socrático do pensamento dialético incluída na esfera do apodítico. Isso sugere que uma alternativa ao racionalismo monístico da metodologia matematizada de Descartes é possível para uma pedagogia em geral, toda vez que ela ponha em jogo novamente o que Descartes encontrou arduamente, isto é, a própria filosofia dialética, a investigação dialógica, a história, a literatura e as artes. Se Sócrates, o qual chegou à filosofia sobretudo como um professor fascinado pelos paradoxos intelectuais, teria explorado, no *Menon*, a possibilidade pedagógica de uma aproximação dialética à matemática, seu trabalho mais amplo na pedagogia dialética poderia ter estado menos centrado no professor, apesar de sua "ignorância irônica"; e dada sua significação histórica como o professor mais reconhecido da cultura ocidental, a pedagogia moderna poderia ter sido menos massacrada com o impulso cartesiano através de soluções únicas, mecanizadas e reducionistas, bem como através de um ensino e um pensamento não comunitário.

Palavras-chave: metodologia cartesiana; método matemático versus método dialético; educação completa; Feyerabend

PHILOSOPHICAL METHODOLOGY AND THE MATHEMATIZATION OF PEDAGOGY FREEING CHILDREN'S IMAGINATION THROUGH PHILOSOPHY

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Socrates: Do we not know that all this is no more than an introduction to the main theme which has yet to be learnt? Surely you would not regard experts in mathematics as masters of dialectic?
Glaucon: Certainly not, except for a few of those I have met.
Socrates: Well, can the knowledge we are demanding ever be attained by people who cannot give a rational account of their statements or make others give an account of theirs? (Plato. *Republic*, vii.532.)

At Trinity and later, in a wide experience of educational problems at London University, I had the advantage of general discussions. It was precisely the kind of education that Plato approves. Mathematics must be studied; philosophy should be discussed. (Alfred North Whitehead. *Dialogues.)*

Philosophy for Children and the controversy over philosophic method:

Pedagogy in Western society is rooted in a notion which evolved largely in the 17th century as a reaction to Plato's attempt to strictly distinguish philosophic and mathematical methodologies and, at the same time, to the flamboyant speculations of Medieval Scholasticism. This reaction, most clearly formulated in Descartes' *Discourse on Method,*[1] made mathematical methodology

[1] The inspiration for the *Discourse,* as Descartes notes, is a complete dissatisfaction with his university education; it is largely as remediation for the misleading tendencies of the educational currents of his time that moves him to write. Concerning the difference between mathematics and philosophy, Descartes clearly prefers mathematical methodology as the vehicle for determining truth: "…in Mathematics there are the subtlest discoveries and inventions which may accomplish much, both in satisfying the curious, and in furthering all the arts, and in diminishing man's labour….Philosophy teaches us to speak with an appearance of truth on all things, and causes us to be admired by the less learned." More specifically: "I was delighted with Mathematics because of the certainty of its demonstrations and the evidence of its reasoning…" In Philosophy, though cultivated by the best minds, "no single thing is to be found in it which is not subject of dispute,

the supreme method of inquiry and, in the centuries that followed, the litmus test for determining what constitutes acceptable academic subject matter; the sufficient reason provided for this claim was that mathematical methodology is simply the best vehicle for engaging and developing thinking; however, in the years that followed, competence in such thinking required children to be subjected to a long stretch of imitative learning—what Thomas O'Brien has referred to as "parrot math." Mathematics, long thought of as providing the model intellectual methodology, has left generations of children with the sense that the intellectual life consists of an endless expanse of arid rules and exercises; the free exercise of imagination was expendable within the school context. The oft' experienced imbalance, within the context of schooling, between tendencies toward fanatical resistance and erratically expressed freedom, when examined, could be traced back to the mathematization of school subjects—except, of course, physical and social play which has always served as the relief valve for many children.

There are obviously advanced areas of knowledge for which mathematics is an indispensable tool, but that is generally not the reason contemporary institutions of higher education require entrance examinations which consist in large part of exercises in mathematics; a majority of students taking those examinations do not move in the direction of the sciences. The main reason, notwithstanding Plato's reservations, is that such tests are supposed to determine the capacity of candidates to engage in critical and abstract thinking. One may note however that, with the exception of Descartes' work in analytic geometry and his invention of the Cartesian system of coordinates, and Spinoza's geometrized Ethics (though Spinoza is remembered more for his philosophic pantheism than for his use of the geometric method), celebrated philosophers have generally not been contributors to mathematics—an observation which

and in consequence which is not dubious….And also, considering how many conflicting opinions there may be regarding the self-same matter, all supported by learned people, while there can would not have surprised Descartes. It is in the last section of Kant's *Critique of Pure Reason* that we get a concentrated effort to provide an explanation for this remarkable fact.

Though Plato gave geometry a place of importance in the curriculum designed for his Republic, its dependence both on propositional knowledge[2] and empirical content left it far behind as a vehicle for the difficult "ascent" (*aporia*) to substantive Formal Knowledge. Plato's characterization of Philosophy as the infusion of "perfect freedom" into thinking process, a process that is broad enough to make possible the non-dialectical logic of mathematics, gave dialogical/dialectical thinking a distinctive property**:** A self-corrective capacity which allows for objectives that exceed the more restrictive boundaries of mathematical thought. Philosophy, in essence, cannot be formulaic.

Dialogical/dialectical thinking, thinking with "perfect freedom," for Plato, could not simply be an application of the methods of the geometers to all questions of knowledge, notwithstanding the fact that, as methods of thinking, there would have to be some connection between these fields. In the 18th century, Kant argued that "freedom" is a transcendental condition for thought in general, and he did this precisely to free thought from its mathematical constraints. "Noumenal freedom," a freedom tied up with the "transcendental (ego) subject," made freedom a constitutive part of all thinking, and unfettered moral deliberation a possibility.

Writing on Kant, Findlay states:

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never be more than one which is true, I esteemed as well-nigh false all that only went as far as being probable." (Descartes, 5ff.)

[2] Plato's interest in "non-propositional knowledge" is explored in F.J. Gonzalez, Dialectic and Dialogue: Plato's Practice of Philosophical Inquiry. Plato's elaboration of the inadequacies of mathematical methodology for philosophic inquiry is discussed at length in Collingwood's Philosophical Method. Plato finds that mathematical method falls short of philosophy's interest in "idealization;" its defects are not open to "remediation;" it fails at "epistemic understanding of its objects;" and it depends on "images." Referring to Plato's famous allegory, Gonzalez writes: "Those who have been outside the Cave and have returned are said to have gained, not proofs nor an axiomatic system, but infinitely greater understanding (vision) of the things within the Cave…" (Gonzalez, 377n 96)

> we have a sufficient ground when we attribute a man's use of A rather than B to his use of freedom, even though we then have no determining or epistemic ground. When we know that a man freely chose to do A rather than B, though he ***could*** have chosen either, we have an entirely adequate explanation of his action, which imports lucidity, not confusion, into our view of the world. This very illuminating conception of the spontaneous as a species, not a violation, of causality, is of course one that Kant employed in his conception of noumenal freedom, even though the conceptual terror inspired in him by Newtonian physics made it impossible for him to hold to it in his conceptions of phenomenal nature.

Continuing his statement, Findlay notes:

> But a law is senseless unless it can be implemented, which brings in the freedom of the elective will. Beings endowed with a will cannot but think of themselves as able to implement its policies, and also by default, not to do so. The possibility of such freedom cannot, however, be illustrated phenomenally, and hence not ***known*** by us to exist. (Findlay, 61, xxi,)

Whether what we have in Kant's transcendental idealism is a set of *a priori* constitutive conditions for granting the availability of philosophic thought to children depends on the extent to which a child's cognition is subject to the same transcendental freedom and "unity of consciousness" which Kant grants to all agents of knowledge. And if it can be demonstrated that imaginative thinking requires those same conditions, children may, *a fortiori*, enter the world of reasoned discourse. In his *Way to Wisdom*, Karl Jaspers provides an empirical confirmation of what Kant could easily explain on transcendental grounds: "A marvelous indication of man's innate disposition to philosophy is to be found in the questions asked by children. It is not uncommon to hear from the mouths children words which penetrate to the very depths of philosophy."

Mathematical method and the advent of pedagogical "reductionism":

Descartes' 17th century reaction to the flamboyant dialectical adventures of the Medieval Schoolmen set the basis for the mathematization of all major fields of knowledge, and, hence, of all "modern" curricula, pedagogy, and pedagogic training. In his work on philosophical method, Collingwood attempts to trace this progression historically. He cites Plato's interest in mathematics as a vital field of study and indicates that "Socrates had found in mathematics a model for dialectical reasoning," but he nevertheless argues that Plato's

> …theory of method must be admitted defective through failure to drive deep enough the distinction established by himself between philosophy and mathematics. The result is that he splits philosophy into two parts: one an arid waste of ingenious logic-chopping, the other an intuitive vision of ultimate reality. ***Descartes***, disgusted with the dialectic of the [Medieval] schools, ***went back to the same model***, and described the lessons he learnt there under four heads: the canons of evidence, division, order, and exhaustion. Nothing was to be assented to, unless evidently known to be true; ***every subject-matter was to be divided into the smallest parts, each to be dealt with separately; each part was to be considered in its right order, the simplest first; and no part was to be omitted in reviewing the whole.*** It was from the study of mathematics that Descartes learnt these rules, and it was to the advancement of mathematics that he first applied them; but he hoped from the first that they would prove useful in a far wider sphere, and by degrees he applied them to the whole field of knowledge as he conceived it: that is, not only to mathematics but to metaphysics and the sciences of nature; for divinity he ruled out as a matter of faith, poetry he considered a gift rather than a fruit of study, and history he regarded as a pastime full of interest and not devoid of profit, but very far from the dignity or utility of a science. (Collingwood, 1933, 16ff. Italics added.)

Plato's critical treatment of the relationship of philosophy to mathematics was lost in later efforts to restrict the unbridled freedom implicit in dialectical thinking. Descartes, in need of a way to make mathematics methodologically applicable to knowledge determinations in general, found in the Scholasticism that he criticized, a procedure which, largely due to his influence, was soon identified as "mathematical methodology." This method (outlined above by Collingwood), largely because it was a step-by-step procedure, had an enormous impact on educational theory and pedagogy. Dialectical adventures into "possible worlds" of the imagination, promoted by some of the great philosophers, yielded to the promotion of sets of reductive skills. This reductionism, which seemed to square with Newton's mechanistic thought, inspired the Enlightenment efforts of the 18^{th} and 19^{th} centuries to revive dialectical reason as a driving, evolutionary force. But earlier, the 16^{th} and 17^{th} century post-Renaissance enchantment with a "machine" model of nature gradually led to the mathematization of pedagogy, a practice which still prevails. In the 18^{th} and 19^{th} centuries, transition from the machine model to a "history" model of nature gradually led to the reconceptualization of pedagogy as "historical" process; this transition, grounded in a deep regard for non-reductionist temporality, served as the foundation for a new concept of education, viz. the "Progressive Education" movement of the 20^{th} century. (Cf. Collingwood, 1945.)

The Progressive response to Cartesian methodology consists of a general critique of the impact of "rationalism" on education: Dewey's critique of Descartes' dualism is clearly applied to pedagogy in *Democracy and Education;* applied appropriately, from the standpoint developed here, in his treatment of "The Nature of Method":

> …under the influence of the conception of the separation of mind and material, method tends to be reduced to a cut and dried routine, to following mechanically prescribed steps. No one can tell in how many schoolrooms children reciting in arithmetic or grammar are compelled to go through, under the alleged sanction of method, certain preordained verbal formulae. Instead of being encouraged to attack their topics directly, experimenting with methods that seem promising and learning to discriminate by the consequences that accrue***, it is assumed that there is one fixed method to be followed***…. Mechanical rigid woodenness is an inevitable corollary of any theory which separates mind from activity motivated by a purpose. (Dewey, 169f. Italics added.)

Following Dewey, in a contemporary critique of reductionist approaches to teaching literacy, Kieran Egan argues that the typical pedagogical "method" used in the teaching of reading sacrifices "imagination" on the altar of mechanistic coding and decoding processes:

> Crude literacy tests often miss the subtle problem which literacy has left us. They count as unqualified successes many cases where students can manage the coding and decoding skills that open the big front door of literacy's storehouse without being equipped to go into the further rooms where its great delights and power are accessible. (Egan, xv.)

Citing recent international literacy tests as an example, Egan points out that the British attempt to drive up literacy scores has not affected the "voluntary reading" rates of British children. "Thus many countries that now score worse than Britain in basic literacy have much higher proportions of children who actually read for pleasure." (Egan, xv.)

Matthew Lipman's important contribution to this discussion involves the elaboration of the notion that children's "reasoning" is precisely what's missing in contemporary literacy programs. The Philosophy for Children curriculum is offered as a remedy. "When we try to teach children to read," Lipman states, "we tend to overlook how mechanical are our techniques, such as those that stress grammar and phonics, and how close these techniques are to what actually blocks the children from reading." (Lipman, 101.)

Descartes' effort to impose a "Scholasticized," reductive method of mathematization on knowledge determinations, when applied to pedagogy, had several lasting results: of all of these results, one stands out as most influential in modern pedagogy, viz., the conceptual confusion of "concreteness" with "simplicity" which infected the teaching-learning process with the cognitive-sensory clutter that has served to limit the intellectual freedom which children exhibit when exposed to dialogue. The corollary notion that "abstraction" is synonymous with "complexity," and that all pedagogy should therefore move from the simple to the complex, that is from the concrete to the abstract, is precisely the reductionist method which is defied by the dialogical thought of children. A contemporary Philosophy for Children curriculum, by returning to Plato's notion that dialectic has larger purposes than mathematical methodology can support, could be a significant remedy for the mathematization of pedagogy which pervades contemporary education and learning theory. The fight is simply against a pedagogy conceived and carried out as a process of "machine" construction and implementation, a process which leads to the fragmentation of both knowledge and the learning experiences of children.

Thus, notwithstanding John Dewey's criticism of the commonplace tendency in contemporary pedagogy to make the knowledge relation ubiquitous by equating concepts with perceptions, contemporary pedagogic texts tend to claim that "concepts" so generated have the sort of "apodicticity" generally reserved for mathematical intuitions. The result comes in the form of a reductive concept of "experience," a "naïve empiricism," and the vague claim that experience is veridical, that "experience teaches"—and teaches correctly!

The pedagogical outcome of this ironic effort to crystallize what is considered "experiential" and empirical is the reduction of children's thinking to a meager, but endless, diet of "simplistic, fragmented materials" from which children supposedly construct "clear and distinct ideas." Turning sense data into "axiom pillars" in the process of concept formation, an ironic "rationalization" of empiricism, is most notoriously evident in such 19th century pedagogic methods as Herbartianism, a not-so-subtle attempt to mathematize teaching methodology along Cartesian lines.[3]

Since much instructional methodology, including mathematics instruction, revels in this confusion of "concreteness" with "simplicity," the result is an "additive" approach to learning which provides children with the mere end

[3] "Principle XXX" in Descartes' "The Principles of Philosophy" announces that "…all that we perceive clearly is true…" Our God-given "faculty of knowledge…can never disclose to us any

products of knowledge and makes knowledge a product of learning rather than learning a by-product of knowledge determinations. Teachers who engage children in dialogue soon notice that, for children, the experience of "concreteness" is actually an experience of the enormous "complexity" of the world around them. Their curiosity seems insatiable. Their job is much more one of interpretation of the world which surrounds them than the mechanical construction of concepts from simplified experiential intuitions—from sensory "axioms." Dewey's attempt to connect concept development to hypothesis formation (*How We Think*) was an attempt to fight this mathematized version of empiricist epistemology. What is being characterized here as the negative impact of the mathematization of pedagogy is underscored in Collingwood's attempt to specify what is limited and restrictive in mathematical methodology:

> Mathematics and dialectic are so far alike that each begins with an hypothesis: 'Let so-and-so be assumed.' But in mathematics the hypothesis forms a barrier to all further thought in that direction: the rules of mathematical method do not allow us to ask 'Is this assumption true? Let us see what would follow if it were not.' Hence mathematics, although intellectual, is not intellectual *a outrance*; it is a way of thinking, but it is also a way of refusing to think. In dialectic we not only draw the consequences of our hypotheses, but we recollect that they are only hypotheses; that is, we are free to 'cancel the hypothesis', or assume the opposite and see what follows from that. (Collingwood, 1933. 13f.)

It is Kant, Collingwood points out, who tries to provide a corrective to the Cartesian view of mathematical and philosophical methodologies. Kant's work provides insight into the characteristics of philosophical methodology, as Collingwood notes; however, since philosophy is defined as the "freedom to think," Kant sees fit to merely inventory the negative aspects of mathematization that limit intellectual freedom. But Kant's task is twofold: he also needs to contain the capricious uses of Pure Reason; and if we add to this his profound

object which is not true…. inasmuch as it apprehends it clearly and distinctly." (Descartes, 270, et respect for phenomenal experience, he might earn the arguable distinction of being one of the first philosophers to view philosophy itself as a cure for extravagant intellectual "methodization."

> [Kant's] aim is not so much to controvert but rather to correct Descartes, by a careful distinction between philosophical and mathematical thinking. He argues in detail that, of the special marks of mathematical science, not one is to be found in philosophy, and the adoption of mathematical methods there can do nothing but harm. Philosophy knows ***no definitions***; or rather, their place in philosophy is not at the beginning of an inquiry but at the end; for we can philosophize without them, and if this were not so we could not philosophize at all. Philosophy knows ***no axioms*:** no truths, there, are self-evident, any two concepts must be discursively connected by means of a third. Philosophy knows ***no demonstrations*:** its proofs are not demonstrative but acroamatic [i.e., esoteric—only for the initiated]; in other words, the difference between mathematical proof and philosophical is that in the former you proceed from point to point in a chain of grounds and consequents, in the latter ***you must always be ready to go back and revise your premises*** when errors, undetected in them, reveal themselves in the conclusion. (Collingwood, 1933. 22ff. Italics are added.)

If Kant's interpretation of methodology is being correctly characterized by Collingwood, and if it can be empirically demonstrated that children can think philosophically (as Matthew Lipman has attempted to do), then Piaget's developmentally restrictive notion that formal thinking requires a capacity to reverse operations would have to be qualified so far as children thinking philosophically is concerned: Reversibility in mathematical thinking would be strictly linear when compared with the "free-ranging" work of philosophy, a work caught up with the determination rather than the elaboration of pre-suppositions; philosophic thinking would serve to loosen the imagination to do its formidable intellectual projections uninhibited by restrictive "stages of development."

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passim.)

But even Kant, in the end, due to his involvement with "transcendental objects," failed to carry forth the critical work of the imagination into an elaboration of metaphysics. Nevertheless, as Collingwood points out, "Kant solved rightly the problem which Plato had solved wrongly, the problem of the methodological difference between philosophy and mathematics, and so laid a firm foundation for all future inquiries into the nature of philosophical method." (Collingwood, 1933. 25.)

Kant's final section of his *Critique of Pure Reason*, "Transcendental Doctrine of Method," is reserved for the study of methodology: The first section of this intense analysis is the source of Collingwood's discussion of the differences between philosophic and mathematical methods. Perhaps the most significant idea which Kant unfolds, an idea which strikes at the very heart of the mathematization of pedagogy, is that philosophy must sacrifice the infallibility which mathematics provides (Descartes, of course, agreed). Hardnosed Behaviorist pedagogues, who, on the basis of "science," claim the infallibility of behavioral learning processes, are paradoxically governed by a philosophy of learning which provides a rather mechanized version of the reflex arc concept. (Both for Descartes and Hegel, the intrinsic connection between Mathematics and Mechanics lends itself to the mechanization of whatever is colored by mathematical methodology—i.e. wherever "quantification" reigns.) Technically speaking, a true Behaviorist, if we applied Kant's position, could never "teach" some of our most important concepts, since such concepts forever lack the clarity and distinctness which mathematical "definition" provides. Strict Behaviorists respond by counting such concepts as "meaningless." Kant recognized that the complex "meanings" embedded in human knowledge, which are the subjects of philosophic study, cannot be grasped as products of Descartes' mathematized method. In a footnote to his discussion, Kant states: "In mathematics definition belongs *ad esse*, in philosophy *ad melius esse*. It is desirable to attain an adequate definition, but often very difficult. The jurists are still without a definition of their concept of right." (Kant, 588a)

If the mathematization of pedagogy is responsible for the stresses and heated discussions of contemporary educators, it is because, as Kant noted, of its exclusive reliance on "quantification." Some educators, adopting the mathematical method as the basic instrument of pedagogy, are subject to Kant's criticism: "Current, empirical rules, which they borrow from ordinary consciousness, they treat as being axiomatic." (Kant, 584.) And, paradoxically, because such pedagogic perspectives are not truly grounded in the synthesis of concepts and experience, contemporary educators are ironically prepared to axiomatize one base after another when their views are subjected to empirical, qualitative critique. The effect is the see-saw movements of pedagogy in the history of education.

Kant's theme can be applied broadly to contemporary issues: "*Philosophical* knowledge is the *knowledge gained by reason from concepts*; mathematical knowledge is the knowledge gained by reason from the [*a priori*] *construction* of concepts."(Kant, 577.) Conceivably, this accounts for the success which pedagogical "Constructivism" has had in mathematics instruction. But it also refers back to the Socratic realization that philosophy is best served by "community of inquiry" pedagogy, since "knowledge gained by reason from concepts" requires the kind of dialectic which demands broad confirmation Thus, if philosophy is best served by a community-based pedagogy, so, *a fortiori*, would all other disciplines which involve the elaboration of concepts. Teaching reading as a process of phonic analysis, when contrasted with an approach which makes the quest for "meaning" the core of the reading experience, can be characterized and paraphrased through Kant's language: "Meaningful reading involves knowledge gained by reason from concepts. Mathematized reading involves skill gained by reason from the construction of terms."

Concerning the methodological difference between philosophy and mathematics, though Collingwood recognizes Hegel's debt to Kant on this matter, he fails to highlight the rather strong position that Hegel himself takes. In the rather extended "Preface" to his *Phenomenology of Spirit,* first published in 1807, Hegel's flamboyant language graphically draws the distinction between these two methods; arguing that mathematical method, concerned not with a dialectical involvement with actualities but with things "merely as units," ends up with "an unmodifiable and lifeless fixity" (Findlay, 1958, 56,). Thus speaking of mathematics, Hegel states:

> The *evident* character of this defective cognition of which mathematics is proud, and on which it plumes itself before philosophy, rests solely on the poverty of its purpose and the defectiveness of its stuff, and is therefore of a kind that philosophy must spurn….The *actual* is not something spatial, as it is regarded in mathematics; with non-actual things like the objects of mathematics, neither concrete sense-intuition nor philosophy has the least concern. In a non-actual element like this there is only a truth of the same sort, i.e. rigid, dead propositions. (Hegel, 25f.)

The rest of Hegel's discussion consists of a brilliant demonstration of the limitations of mathematics from the standpoint of cognition. A critique of the most serious "proofs of propositions," which Hegel briefly demonstrates, "would be as noteworthy as it would be instructive, partly in order to strip mathematics of these fine feathers, partly in order to point out its limitations, and thus show the necessity for a different kind of knowledge." (Hegel, 27.)

Before getting to mathematics as a topic in his work on ***Logic***, Hegel distinguishes three methods for "ascertaining truth" and indicates that each of these methods is "no more than a form;" that is, in each instance mind must bring something to that form in order to know truth: (1) ***experience,*** (2) ***reflection***, and (3) the ***pure form of thought*** are the three methods available to man. Philosophic method, or "philosophic cognition" as Hegel calls it, is that ***pure form of thought*** which brings ***perfect freedom*** into inquiry. The first of these, "experience," is the stage of "human innocence;" the other two open an involvement with the binary stresses which subject man to the level of the problematic in knowledge and morality. (Hegel, 1873, 52f.)

If we inquire with Hegel concerning our proximate interest, the impact of giving priority to "the mathematical categories" on pedagogy overall, what he offers in an esoteric passage in his ***Logic*** is a prescient insight, viz. that mathematization introduces materialism**:** and, indeed**,** it is the overwhelming materialization in contemporary pedagogic methodology and its resulting fragmentation of knowledge that is the *bete noire* in this discussion. Given Hegel's notion that mathematization gives priority to "quantification," it is understandable that he would view it as a limited "category of the Understanding" (using Kant's language).

> If quantity is not reached through the action of thought, but taken uncritically from our generalized image of it, we are liable to exaggerate the range of its validity, or even to raise it to the height of an absolute category….Our knowledge would be in a very awkward predicament if such objects as freedom, law, morality, or even God himself, because they cannot be measured or calculated, or expressed in a mathematical formula, were to be reckoned beyond the reach of exact knowledge, and we had to put up with a vague generalized image of them, leaving their details or particulars to the pleasure of each individual, to make out of them what he will…..And this mere mathematical view, which identifies with the Idea one of its special stages, viz. quantity, is no other than the principle of Materialism. (Hegel, 1873, 186f.)

Perhaps the most troublesome effect of the mathematization of pedagogy has been the tendency on the part of educators to operate as if all pedagogical problems have a solution—indeed, a single solution to the exclusion of all other possibilities. This utopian approach to teaching seems to separate pedagogy from all other practices—medical, political, legal. It is time for pedagogy to adopt the first principle of all other practices, and that is, above all, to "Do no harm." The commonplace labeling, sorting, and consigning of children, supported by a reductionist, psychologized epistemology, suggests a profound neglect of this principle, a neglect made evident by the subsumption of the human child to pedagogic mechanization and bureaucratization. Seeing this as a methodological issue, an issue which reflects the larger philosophic question raised here, calls for a form of pedagogic practice which requires "reflective, philosophic *practitioners*, " not mere scribblers of "My Philosophy of Education."

Has reductionist technology served to mathematize children's play?

Is the contemporary child's "wired play" having a negative impact on children's cognitive and social development? Has the loss of the constructional play materials of the past (blocks, Erector Sets, model airplanes, dolls and doll houses) become an impediment to the development of the kind of imagination in children which, if Kant was correct, would support the intellectual freedom required for "critical" thinking? Are children now entering the adult world increasingly involved with passive enjoyment rather than productive work? Having lost the "dialectical instrumentation" of imaginative play, has play given way to linear reductionism and structurally patterned thinking?

Psychology, long given to dialectical conflict, has come down on both sides of this issue. There is the claim that the long stretches of constructive imaging of traditional play, which put children into direct contact with their physical and social worlds, has been lost in the world of TV and computers. But does this notion, identified as the 19th century preference for the "protected child," isolate the child from the kind of play which the contemporary world, into which the child must grow, prefers? Intellectually speaking, is a Philosophy for Children curriculum, built on dialogical interchange, an anachronism? And is that the reason why many teachers find such a curriculum irrelevant? Were the Greek rhetoricians, who were attacked by Socrates on methodological grounds, correct after all—shouldn't teachers prepare children to win at the social games played in the world that surrounds them? But this is not a problem for mathematized methodology to resolve; it is, in the end, the "perennial problem of philosophy." Dewey's "pedagogic creed" put it succinctly: the conservatism of one period is the liberalism of an earlier period. The Erector Sets of the mid-20th century were still home-bound and did not activate the growing child more than the farm work or factory job of an earlier period. The problem of philosophy is to guide each era to re-institute the conditions for the development of dialogue between the child and his or her world; so must it be for the electronic age. As philosophy has been historically, so a Philosophy for Children curriculum can become the overarching educational enterprise which takes up the challenges of each era and connects them to the free and imaginative thinking of children.

In the end, if Kant and Hegel are correct, it would not be surprising to hear some disquieting thoughts of teachers concerning the matter under discussion: In the general context of education, students of mathematics are not necessarily better thinkers or writers than other students; furthermore, students who claim that they were never really successful in their mathematics studies often turn out to be good and productive thinkers in a variety of fields. Nevertheless, from mechanical approaches to lesson plans and texts; from lectures to course outlines; from the concentration on patterned and repetitive learning rather than imaginative/dialogical thought; the reductionist formalization of instructional process and learning practice has been the effect of a utopian mathematization of pedagogy—a pervasive effort to provide teachers (and, ultimately, students) with a uniform and universally applicable methodology for overriding all of the "qualitative difficulties" inherent in so complex a practice as teaching, learning, and intellectual problem-solving.

The historical reason for this contemporary utopian interest in pedagogical reductionism; the reason that mathematization has given pedagogy an air of certitude—a certitude based on what Hegel called "rigid, dead propositions," is simply because pedagogy has never given up the classical interest in "completeness," in what the 19th century referred to as "complete education." Because Plato could not achieve a vision of such completeness; because the formal world of Ideas eluded him and could not ultimately be completely captured and integrated with the experiences in the Cave; for that reason alone Plato settled for an ongoing dialogical pedagogy which celebrated the moral significance of the process itself—what Plato called "protrepein" (*Euthydemus)*. The "protreptic" teacher needed to achieve the moral stance which was characteristic of dialogical pedagogy—a communal "turning toward" truth which was already infused with the Good and the True, even if incomplete. If Descartes thought he could achieve "complete education" through the mathematization of knowledge, complete in spite of the failure of the derided Medieval schoolmen to do so, he ended with as many unanswered questions as did the Socratic teacher who thought it was only possible to do so through dialogical/dialectical inquiry. Though Dewey sought to resolve this issue, which he inherited from 19th century philosophy, by the pedagogic promotion of the more modest "complete act of thought;" though Hegel fought to find a solution through a thoroughgoing idealization of reality, and Marx through the materialization of Hegel's "notions," contemporary teachers are still pursuing the utopian ideal of complete education, but now through a mechanized reductionism, a naïve confusion of "teaching" with "telling," knowledge with simplified sense experiences, meaning with reference, and the "possible worlds" of the imagination with facticity.

By far, the most criticized tool of reductionist teaching is the modern textbook:

> But strange as it seems, these books just scratch the surface, and that's because they contain *too much* material….In the drive to include everything, key ideas fade into the background, or are never successfully communicated, or simply don't stick with students. (Daniels and Zemelman, 39.)

Mathematical "Methodization": The de facto business of contemporary schooling:

It needs to be noted, in passing, that Hegel's ***Logic*** did not involve a wholesale disparagement of mathematics or a rejection of the significance of mathematical method. He simply wanted to point out that "quantity," which is the category of mathematical inquiry, is a "stage of the Idea" (a concept which needs to be read in the context of Hegel's "Idealism"). Its importance for the natural as well as spiritual worlds is underscored in Hegel's philosophy. But "to seek all distinction and determinateness of objects merely in quantitative considerations" is "in the interest of exact and thorough knowledge, one of the most hurtful prejudices." Contextualizing our knowledge of the world around us provides us with the "qualitative" insights which serve our knowledge and our capacity to individuate actualities (Hegel, 1873,188.). Notwithstanding this Hegelian position, the contemporary pedagogic concentration on "skills"—even skills of "critical thinking"—in isolation from practice has been the result of a philosophic inattention to the qualitative difference between "practice" and "exercise," a difference swallowed up by an indifference to the mechanically fixed or completion-driven condition of school mathematics which then serves as the idealized model for all other fields of study. Furthermore, the methodological significance of "counterinduction" and "unsupported hypotheses" for the qualitative thought contained in fields of practice gets completely overlooked. That indifference, when generalized to all forms of practice, is responsible for that negative flavor of "abstractionism" which children often identify with schooling. The seeming apodicticity of school learning is bought at the expense of a serious awareness of the limitations of theoretical perspectives. It is, therefore, not surprising that some scholar would, like Descartes himself, once again pick up the weapon of intellectual anarchism, and once again find that methodology is the culprit. Nothing could say this better than Paul Feyerabend's book title: *Against Method*, which unfolds a critical examination of the place of method in the history of philosophy and science, and which needs to be read as a corrective for pedagogy as well as science and philosophy:

> Not only are facts and theories in constant disharmony, they are never as neatly separated as everyone makes them out to be. Methodological rules speak of 'theories', 'observations' and 'experimental results' as if these were well-defined objects whose properties are easy to evaluate and which are understood in the same way by all scientists. (Feyerabend, 51)

Feyerabend's critique of the general "quest for certainty" is not only reminiscent of Dewey's work, but it brings us back to the beginning: Descartes' *Discourse on Method;* but even here, in the one place in modern philosophy where "certainty" seems assured because reality is ultimately open to mathematization, Descartes utters some last minute reservations:

> I must confess that the power of nature is so ample and so vast, and these principles [the theoretical principles he had developed for his mechanical universe] so simple and so general, that I almost never notice any particular effect such that I do not see right away that it can [be made to conform to these principles] in many different ways; and my greatest difficulty is usually to discover in which of these ways the effect is derived. (Cited in Feyerabend, 49n.)

Fearing the possible appearance of intellectual anarchism, Feyerabend raises the obvious question: Does the attempt to restrict mathematization in the form of "rationalist" methodology, whose inspiration has always involved what Hegel felt was a process that loses the "deeper affinities or relations" that things have with one another (Findlay, 1962, 56), move us in the direction of intellectual and cultural relativism? In the context of educational thought, the promotions of multiple, exclusivistic philosophies and philosophies of education have created that sense of foundational relativism which has turned teachers away. The result has been the adoption of the utopian alternative which mathematization promises in the form of a single solution to all aspects of pedagogy. A single-solution pedagogy, then, produces what Feyerabend refers to as a "guided exchange" in contradistinction to an "open exchange." (At a much earlier time, Dewey distinguished between an "apprenticeship" approach to teaching and a "laboratory" approach. It is well known which of these he preferred.) Citing one of his earlier books, Feyerabend states:

> A guided exchange adopts 'a well-specified tradition and accept[s] only those responses that correspond to its standards. If one party has not yet become a participant….he will be badgered, persuaded, 'educated' until he does—and then the exchange begins.' 'A *rational debate*', I continue, 'is a special case of a guided exchange.' In the case of an open exchange 'the participants get immersed into each other's ways of thinking, feeling, perceiving to such an extent that their ideas, perceptions, world-views may be entirely changed—they become different people participating in a new and different tradition. An open exchange respects the partner whether he is an individual or an entire culture, while a rational exchange promises respect only within the framework of a rational debate An open exchange has no organon though it may invent one; there is no logic though new forms of logic may emerge in its course.' In sum, an open exchange is part of an as yet unspecified and unspecifiable practice. (Feyerabend, 269.)

Pedagogy has for too long a time been distracted by the quest for "***the*** philosophy ***of*** education" which will serve all traditions and individuals, and this search, which reflects Feyerabend's "rational debate," has often been accelerated in the context of a pluralistic society. But what Socrates first pursued, and what we can infer from Feyerabend's discussion, is the need for a "philosophy ***in*** education" pedagogy—the very pursuit which a Philosophy for Children curriculum would underwrite. Here "good teaching" can be redefined: Using Robert Nozick's terminology, one consistent with Feyerabend's discussion, good teaching strives for a "value-theoretic situation" rather than a "game-theoretic situation": It strives for the dialogical transaction which constitutes the "open exchange" recommended by Feyerabend as an alternative to both absolutism and relativism**—**the two contentious extremes which have always emerged as the result of the mathematization of method; it strives for an acknowledgement of the value of the minds of all children and teachers as members of a dialogically operational "community of inquiry."

The philosophic significance of this move "against method" is that it undercuts all of those positions which generate absolutism/relativism controversies—however subtle: Instead of raising to a philosophic level the question of the relevance for pedagogy and pedagogical methodology of such prescriptive concepts as Howard Gardner's concept of "multiple intelligences," a notion that psychologizes learning and gives priority to Hegel's least significant method for ascertaining truth—the method of experience—the method which "depends upon the mind we bring to bear upon actuality," schools of education turn these into the definitive "subject matter" of pedagogic study. The fragmentation in pedagogy which results from this, Hegel would have argued, must ultimately end in scepticism or finally be overcome by the philosophic approach to knowledge which, paradoxically, involves the "attitude…of entire freedom" in the quest for "absolute truth." Hegel's formalism, evident in his notion that these methods for ascertaining truth are "only forms," was an effort to avoid the sort of reductionist fragmentation and pedagogic scepticism which plagues contemporary education. Gardner's view, as an example, is just one way of concretizing the experiential form of methodology; there are obviously others which contradict or even totally bypass Gardner's schema. Hence, this view provides students of pedagogy with still another one of those experiential formalizations which they will drag into their classrooms—and very quickly neglect or forget: Piaget, Maslow, Skinner, Bruner, et al., are still others. Approaches to pedagogy which give axiomatic status to one or other experiential concretizations fail to see the "logical" (in Hegel's sense) limitations in the use of the "method of experience"—the method which reifies and projects the "innocence" which comes in the form of "immediate knowledge."

Once dialogical/dialectical "reflective and philosophic cognition" are removed from the work of the pedagogue, that is, once "reflective cognition" is sacrificed in the face of the quest for "*the* method," the exigencies of practice soon test the "innocence" which the pedagogic student brings to the field, an innocence which remains uncontested in his professional training; in fact, it is the ultimate source of the teacher's pessimism which sooner or later makes its way to consciousness: The failure of the promise of methodology ultimately becomes unforgiving. Looking through Hegelian eyes, one can see that contemporary teachers haven't abandoned the pursuit of the domain of innocence—of "immediate cognition" given "absolute" status, a notion which becomes inhibitive to the freedom implicit in "reflective and philosophic cognition;" that is, they have not yet faced the critical question: What if Feyerabend is right? What would happen if teachers were "against method?" What would be left to do? Would teachers do what they were taught to do—turn the "against method" disposition into a *new* method? In the *Phi Delta Kappan* (1998, 2000), Martin Bickman asked for the recuperation of the "tradition of the active mind for teacher education." His plea was then, as it is now, germane to this discussion: "Educational structures are more in flux now than they have been in recent memory, and before they freeze into new rigidities and simplicities, there may be a chance to restore thinking—the continual act of mind—as the central activity of schooling." He cites John Dewey's concern that his own point of view had been converted by teachers colleges "into a fixed subject matter of ready-made rules, to be taught and memorized according to certain standardized procedures and, when occasion arises, to be applied to educational problems externally, the way mustard plasters, for example, are applied."

Hegel said it almost two centuries earlier: "The two other forms [other than the "method of experience"], reflective and philosophical cognition, must leave that unsought natural harmony behind" (Hegel, 1873, 53). The inadequacy of innocence, which is unaddressed in professional pedagogical studies, can be seen in those statements by students of pedagogy which are notoriously paradoxical—for example such commonplace statements which romantically claim that it is "absolutely true that no two children know things or learn things in the same way," and that "no two teachers teach alike, since they all have their own methods." But these are the two propositions most clearly violated in contemporary schooling. And all of these statements are nurtured by the most outrageous claim of all, viz. that each student of pedagogy has his very own philosophy of education.

We need to underscore these latter observations: That sense which teachers have that their pedagogic training and the actualities of schooling are out of sync is largely due to the fact that their "philosophies of education," so dear to schools of education, are really nothing more than examples of Hegel's "innocent" expressions endowed with the status of certitude by the "method of experience;" these vague generalizations that make up their thinking are not filtered through Hegel's "free use of reflective and philosophic cognition." "Reflective teachers" are the exception, not the rule. The top-down "professionalization" of pedagogy is perhaps the underlying culprit. Schools of education protect themselves by both formalizing and indefinitely extending the innocence of Eden, by letting Hegel's "natural harmony" become crystallized through the reductive "method of [naïve] experience," and by protecting their students from confrontations with the Serpent.

Philosophical methodology embedded in a new pedagogy of mathematics: A challenge to the "single solution" bias in contemporary teaching.

A modest attempt to reverse the deleterious effects of the "machine" model of pedagogy, a mathematized and reductivist pedagogy constructed along lines developed by such philosophers as Descartes, is gradually finding its way into mathematics instruction itself. It is suggested here that this somewhat new development impacts on Philosophy for Children in a special way: It brings the mainspring of philosophic thinking into the one field which has been resistant to engaging philosophically with children and which is the basis of the pedagogy under review in this paper—viz. mathematics. The notion that "conceptual understanding" requires a long process of implanting isolated skills and details onto the "blank slate" which is the mind of the child is now being challenged precisely in the way that Socrates did in his response to Protagoras: Philosophical methodology does not work, cannot work, from isolated part to whole; in fact, Socrates would have argued, any and all methodologies involving the use of reason require a sense of direction which only a reference to the whole—to the "solution," even if only hypothetical—provides. This romance with classical teleology has never died; it lives and thrives in the pragmatism of Dewey and Rorty. In the Socratic method, this sense of the whole needs to be hypothetical; it opens those diverse possibilities which nurture dialogue, and, as such, is philosophically indispensable. The child's eagerness to promulgate and to make declamatory statements convinced Socrates that the child is innately connected to knowledge—that the child has "a mind of its own" which cannot be disregarded pedagogically.

Socrates would have seen contemporary pedagogic reductionism as a return to the position of the Sophists: viewed from the point of view of the "machine" model, the child is initially "blind," cognitively speaking, and is therefore utterly dependent on those who guide him.

Just as in our ordinary lives we act virtuously by being inspired with some awareness of what virtue is, so is our pursuit of the good guided by similar inspiration. The existence this kind of inspiration grounds the theory of education…: education does not, as some people (presumably the sophists) arrogantly profess, introduce knowledge into the mind as sight into blind eyes but rather turns in the right direction a mind already in possession of latent knowledge. (Gonzalez, 211)

The purpose of this move for Socratic teaching is precisely to avoid the reductionism which otherwise makes a mess of the drive toward general concepts.

The virtue of the kind of explanation Socrates advocates here is its avoidance of reductionism.

The explanation of a thing's beauty in terms of its possession of a certain shape or color reduces beauty to something that at best contributes to it. Such an explanation can therefore be easily refuted by citing something that either has a completely different shape or color and is equally beautiful, or has the same shape or color and is not beautiful. (Gonzalez, 200f.)

We do not need to examine critically the five or so historical interpretations of the Socratic position on the "ascent to" and "descent from" general concepts; these are carefully evaluated by Gonzalez. Nor do we need to examine those views of the Socratic position, criticized by Gonzalez, which attempt to collapse "the distinction between methods of mathematics and dialectic, and thus between *dianoia* and *noesis."* (Gonzalez, 221.) But it seems possible to build through Socratic methodology a pedagogic approach to mathematics that might avoid the reductive mechanization commonly experienced by students; it would need to adopt the whole-to-part approach which, if Socrates was correct, governs our thinking in general and our philosophic thinking in particular. This approach avoids both the cognitive dependency which mechanization requires and the "blindness" of the student who finds himself in a forest of particularities.

In recent work on mathematics instruction, Sullivan and Lilburn have moved in this direction. Simply put, they argue that mathematics instruction should involve reflective problem solving rather than mechanical exercises, and to do this the teacher must reverse the standard procedure; this reversal opens the possibility of hypothetical thinking in a field which is typically taught in a cut-and-dried fashion. These mathematics educators give many types of examples, but one should suffice: Mathematics teachers who teach "averaging" will generally provide students with—let us say—the ages of five children, and then ask for their "average age." Students must then simply apply this commonplace procedure: add the ages and divide by five. Let's say the resulting average age is nine. A simple mechanical process produces one possible solution, and no conceptual indication as to what "averaging" means. Sullivan and Lilburn complain that this approach involves the use of "closed questions" and, we might add, these give mathematics that *single*-solution character which Hegel decried and Feyerabend called a "guided exchange" in contradistinction to an "open exchange." The approach which Sullivan and Lilburn recommend would begin with what they refer to as a "good question." Their approach would introduce the same five children, but it would provide the solution by indicating that their average age is nine. The "good question," as in a Socratic dialogue which starts with a proposed definition, would then ask for the discovery of their *possible* ages—i.e. the possible conditions which make the solution possible (Sullivan and Lilburn, 2002). It does not require a great stretch of mind to see the dialogical possibilities, internal or communal, in this procedure. And though it deals with "quantification," this approach might have satisfied even Hegel. Indeed, had Socrates tried it, Meno's servant boy might have had a more conceptually significant mathematical revelation than in Socrates' geometric excursion; furthermore, Socrates' theory of learning as "recollection" would have been more dramatically served by this demonstration. Did Plato's attempt to separate philosophical and mathematical methodologies result in his leaving mathematics in the more mechanistic single-solution domain? Was he so concerned with what mathematics has to offer philosophical thought that he failed to inquire whether philosophical methodology has anything to offer mathematics? That is, pedagogically speaking, did the Socratic-Platonic epistemological alliance not go far enough? In the Sullivan-Lilburn approach, it is not difficult to see the spirit of philosophical methodology plowing new pathways in the garden of mathematics. *Requiescat* Descartes.

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