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STATIONARITY OF EXTREME STREAMFLOW TIME SERIES FOR FLOOD CONTROL OPERATION IN HYDROPOWER RESERVOIRS

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Abstract

The Brazilian Power Sector consider flood prevention in the operation by reserving during the rainy season empty volumes in its hydropower reservoirs. The currently used flood control methodology assumes seasonal stationarity for the naturalized daily streamflow records calculated by the Brazilian National Operator System (NOS/ONS). Considering that climatic and/or land uses and occupation changes may be altering hydrological regime, this article investigates the stationarity of maximum annual streamflow time series of duration compatible with flood events at the hydropower plants located in the Paraná river basin.

Keywords*: Statistical Tests, Extreme Streamflow Time Series Stationarity, Hydropower, Flood Control Studies.*

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1. Introduction

Due to the large hydropower potential, Brazil has hydroelectricity as its main source of electricity, which is responsible for 60.56% of the national installed capacity (ANEEL, 2020).

In addition to electricity generation, the reservoirs of hydropower plants (HPP) of the Brazilian Interconnected Power Systems are also used to control floods. During the rainy season, the Brazilian Power Sector reserves empty volumes in its hydropower reservoirs, called “waiting volumes” (WV), for dampening of potential floods, protecting the downstream valley. The waiting volumes are allocated as curves of weekly empty spaces throughout the rainy season.

The use of HPP´s reservoirs to concomitantly control floods and generate electricity is conflicting, since each use demands the operation of the reservoir in opposite ways. To minimize this conflict, Brazilian National Operator System (NOS/ONS) elaborates the Flood Annual Plan (ONS, 2018) with the objective of calculating the WV curves that will be allocated in the reservoirs along the next coming rainy season. Since, at the WV calculation moment, the next hydrological inflow time sequence is unknown, the problem is treated (COSTA *et al*., 1999; COSTA *et al*., 2014; CEPEL, 2016) in a stochastic way by considering a large number (12.000) of synthetic scenarios of possible daily inflows time sequences. The scenarios are obtained by a multivariate stochastic model (KELMAN *et al*., 1983; COSTA *et al*., 1999) which reproduces the main characteristics of available naturalized daily streamflow time series (~ 65 years), such as seasonality and asymmetry. This generation assumes that the naturalized streamflow records are seasonally stationary.

As the waiting volumes allocation uses scenarios of possible daily inflows generated based on historical streamflow records and, motivated by the hypothesis that climatic and /or land use and occupation changes may be altering the regime of extreme flows, this paper proposes to investigate the behaviour of maximum annual streamflow time series of duration compatible with flood events at a set of hydropower plants located in the Paraná river basin in order to verify the existence or not of non-stationary patterns.

2. Methods for time series stationarity analysis

2.1 General Statistical Tests of Non-Stationarity

Non-stationarity behavior in general time series data can be verified by parametric or non-parametric statistical tests according to the convenience of considering the family of the probability distribution of time series values as known or not known. Usually, tests developed with the normality assumption are used as an approximation without considering the assumption validity. Also, usually some statistical tests are best suited to verify certain types of non-stationarity behaviour, for instance, abrupt (step) or gradual (trend) changes in the time series.

Table 1 shows the set of general statistical tests (MOOD *et al*., 1974) selected for this study:

Table 1: Statistical tests selected to verify non-stationary behaviour in general time series

Test	**Verify/Identify**	**Parametric**
Student's T	Abrupt changes in the average	YES
F-Snedecor	Abrupt changes in the variance	YES
Mann-Whitney	Samples come from the same population	NO
Rho de Spearman	Presence of trend	NO
Cox-Stuart – Sign Test	Presence of trend	NO
Run Test	Randomness of data	NO
Mann-Kendall	Presence of trend in **a single** time series	NO

The tests in Table 1 were applied to each time series separately. In order to complement the study and verify the condition of the entire Paraná river basin, it was applied the multivariate Mann-Kendall test (LIBISELLE and GRIMVALL, 2002; YE et al., 2015), which is a non-parametric test to jointly verify if there is a statistically significant trend in a set of time series.

The multivariate Mann Kendall test can be formulated as:

H_0: The set of time series shows no trend

H_1: The set of time series shows trend

Initially, a statistic S is calculated for each time series: With the time series $\{x_1, x_2, \ldots, x_n\}$, the statistic S is calculated as the sum of the signs of the differences, pair by pair, of all the values of x_i with the values of x_j, where j represents all the positions after i. The Equation (1) explains the calculation of S in the form

$$S = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} sign(x_j - x_i) \quad (1)$$

where

$$sign(x_j - x_i) = \begin{cases} +1, if x_j > x_i \\ 0, if x_j = x_i \\ -1, if x_j < x_i \end{cases} \quad (2)$$

Then the matrix of covariances between all pairs of time series g and h is calculated according to Equation (3) :

$$\Gamma_{gh} = \frac{1}{3}\left[K_{gh} + 4\sum_{j=1}^{n} R_{jg}R_{jh} - n(n+1)(n+1)\right] \quad (3)$$

where

$$K_{gh} = \sum_{1 \le i < j \le n} sign((x_{jg} - x_{ig})(x_{jh} - x_{ih})) \quad (4)$$

$$R_{jg} = \frac{(n+1+\sum_{i=1}^{n} sign(x_{jg} - x_{ig}))}{2} \quad (5)$$

With d the number of time series, the statistic Z is calculated by Equation (6) as

$$Z = \frac{\sum_{i=1}^{d} S_i}{\sqrt{\sum_{j=1}^{d}\sum_{i=1}^{d} \Gamma_{ij}}} \quad (6)$$

After calculating the statistic Z and adopting a significant level, it is possible to reach a conclusion on the joint stationarity.

2.2- Specific Tests of Non-Stationarity for Extreme Values Time Series

General stationarity tests assume that the data values in the time series are independent of each other. In the case of parametric tests, usually the assumption that the data values have a normal distribution is added, whereas in non-parametric tests, a distribution is not specified. When working with extreme events, the data values can be considered independent, but is much more appropriated to replace the normal distribution assumption by asymptotic extreme values distribution.

The exact distribution of an extreme random variable of interest Y, F_Y (y), depends on the distribution of the original random variable X, F_X (x), and on the number of variables X, *n*, used to define Y (NAGHETTINI and PINTO, 2007). On the other hand, if F_X (x) is unknown, F_Y (y) will also be unknown. For these cases, there is the asymptotic theory of extreme values (NAGHETTINI and PINTO, 2007), according to which $\lim_{n\to\infty} F_Y(y)$ belongs to one of the three types of extreme distributions: Gumbel, Fréchet or Weibull, and these three types can be written as a single distribution, called the generalized extreme value distribution (GEV). In general, time series of maximum are characterized by the Gumbel or the GEV distribution. The Gumbel and GEV cumulative distribution functions are presented in Equations (7) and (8) as

$$F_Y(y) = exp\left\{-exp\left(-\frac{y-\mu}{\sigma}\right)\right\};\ -\infty < \mu < \infty\ e\ \sigma > 0 \tag{7}$$

$$F_Y(y) = exp\left\{-\left[1-\xi\left(\frac{y-\mu}{\sigma}\right)\right]^{1/\xi}\right\};\ -\infty < \mu, \xi < \infty\ e\ \sigma > 0 \tag{8}$$

where μ, σ and ξ represent the location, scale and shape parameters. As mentioned, the GEV distribution encompass the three types of extreme distributions in a single expression, noting that the shape parameter ξ defines the type. It is observed that the GEV distribution becomes the Gumbel distribution by taking lim $\xi \rightarrow 0$.

Given a historical time series of Y, $\{y_1, y_2, \ldots, y_n\}$, the estimation of the parameters of any distribution can be done using the maximum likelihood estimation (MLE) (MOOD et al., 1974), which determines which combination of parameter estimates is most likely according to the metric of the product of the densities, f_Y (y), applied to the values in the historical time series (likelihood function). The likelihood ratio test consists of comparing the likelihood functions of two different models, being one of them simpler than the other, to verify whether increasing the level of complexity results in a statistically greater likelihood.

In order to analyze the stationary conditions of maximum annual streamflow time series, the MLE method was used through the R.extRemes package, version 2.0-9 (GILLELAND and KATZ, 2016), to fit the extremes distributions according to four (4) hypotheses:

a. **Stationary series** (f10 model) – The parameters do not vary over/in time:

$$\mu = \mu_0 \tag{9}$$

$$\sigma = \sigma_0 \tag{10}$$

$$\xi = \xi_0 \tag{11}$$

b. **Series with the location parameter varying linearly over/in time** (f11 model) – In this case, a location change rate parameter, μ_1, is estimated. Thus, the location parameter is defined by Equation (12):

$$\mu = \mu_0 + \mu_1 t \quad (12)$$

c. **Series with the scale parameter varying linearly over/in time** (f12 model) - In this case, a scale change rate parameter, σ_1, is estimated. Thus, the scale parameter is defined by Equation (13):

$$\sigma = \sigma_0 + \sigma_1 t \quad (13)$$

d. **Series with the location and scale parameters varying linearly over/in time** (f13 model) - In this case, additional parameters for location and scale change rates are estimated. Thus, the location parameter is defined by Equation (12) and the scale parameter by Equation (13).

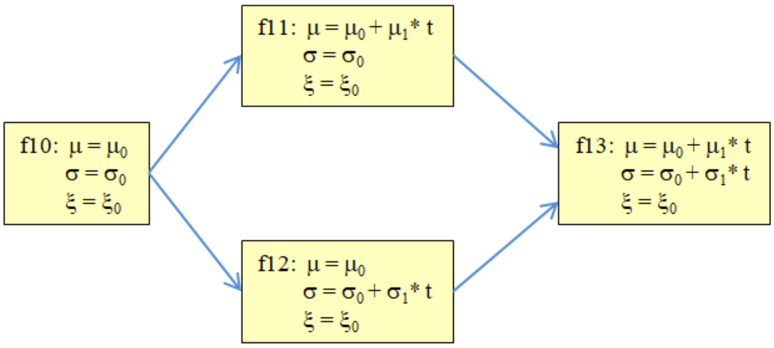
As maximum time series tend to follow a Gumbel or GEV distribution, we choose to fit these two stationary models to the time series of each analyzed time series. The likelihood ratio test (MOOD et al., 1974) was applied to verify which of the models was statistically more likely. Non-stationary fits were made according to the best stationary fit, for instance, if the best stationary fit for the time series was Gumbel, the non-stationary adjustments for this time series also followed the Gumbel distribution (DAMÁZIO and COSTA, 2014). In order to compare the fitted models, the likelihood ratio tests shown in table 2 were applied:

Table 2: Null and alternative hypotheses of the four likelihood ratio tests

Test	Null hypothesis (H_0)	Alternative hypothesis (H_1)	Comparison
i	Stationary	Non-Stationary on location	f10 x f11
ii	Stationary	Non-Stationary on scale	f10 x f12
iii	Non-Stationary on location	Non-Stationary on location and scale	f11 x f13
iv	Non-Stationary on scale	Non-Stationary on location and scale	f12 x f13

By analyzing the results of the likelihood ratio tests, it was determined whether each maximum annual streamflow time series is stationary or not and, if not, which hypothesis is the best. The diagram in Figure 1 shows the sequence of the tests' application. It is worth noting that the f13 model should only be tested against non-stationary models (f11 and f12), if at least one of these models has statistically greater likelihood than the stationary model.

Figure 1: Schematic diagram of the stationarity tests' sequence.

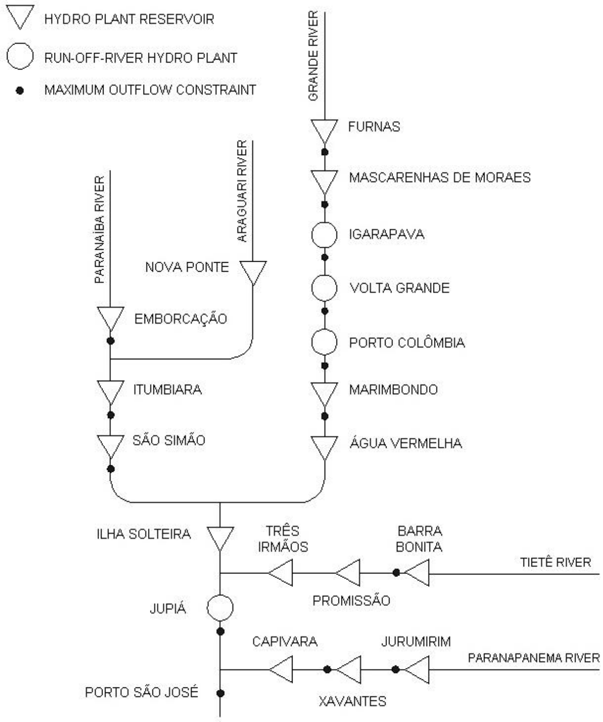


3. Case Study

The choice of the Paraná river basin was not only due to its importance in terms of electricity generation, but mainly because it is the largest basin in Brazil that performs flood control. We have considered in this study only the portion of the Paraná river basin upstream from Porto São José fluviometric station. For this sub-basin the rainy season occurs from November to April and the flood control system comprises 14 flood control points with maximum outflow constraints and 14 reservoirs of HPPs (Figure 2).

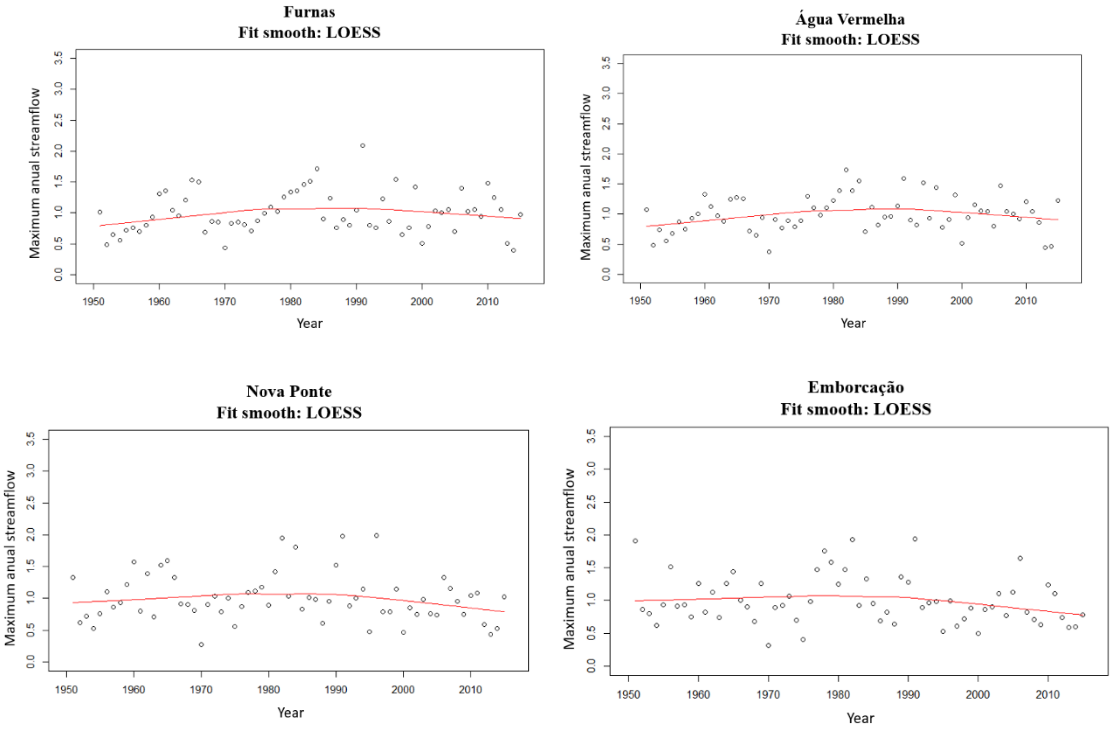
We selected data from HPPS fluviometric stations located upstream and downstream of the rivers that form the Paraná river, which resulted in ten HPPs: Furnas, Água Vermelha, Nova Ponte, Emborcação, São Simão, Barra Bonita, Promissão, Jupiá, Jurumirim and Capivara. The daily streamflow records from 1951 to 2015 used in the studies of the Flood Annual Plan - Cycle 2018/2019 were obtained from ONS. Since the focus of this study is flood control, and a flood lasts longer than a single day, it was decided to consider time series of annual maximum of 10-day average streamflow. Annual maxima were calculated considering the hydrological year starting in November. For illustration the graphs in Figure 3 shows the calculated time series standardized using the division by the average together with trend curves fitted using local regression technique.

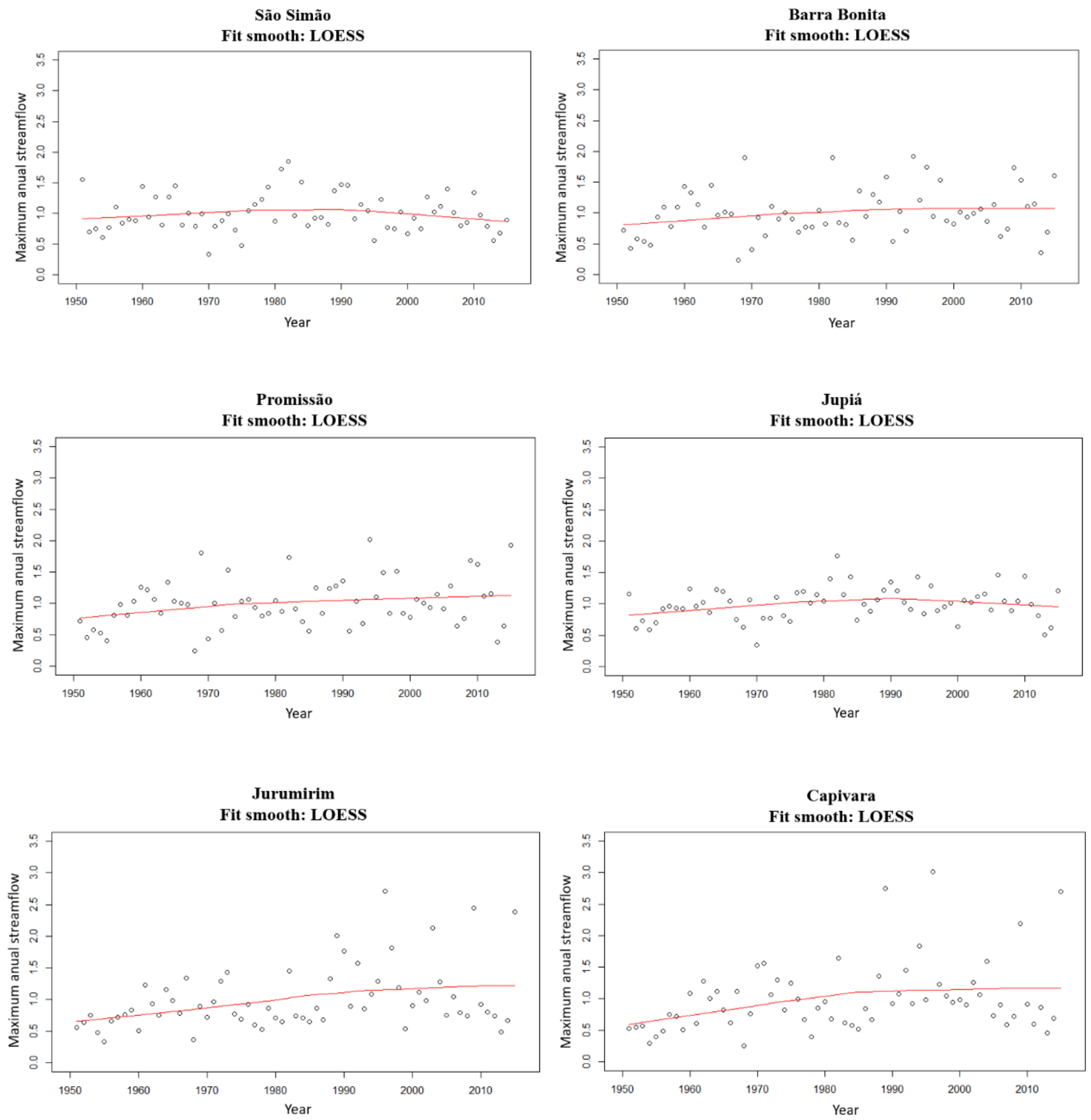
Figure 2 – Topology of the Paraná river basin upstream Porto São José fluviometric station.



Fonte: Costa *et al*. (2014).

Figure 3 – Standard 10-day average maximum annual streamflow





4. Results

Tables 3 to 5 present the results obtained in each general statistical test selected for this study.

Table 3 – Results of Student's T, F-Snedecor and Mann-Whitney tests.

HPP	STUDENT'S T TEST		F-SNEDECOR TEST		MANN-WHITNEY TEST	
	Statistic T	p-value	Statistic F	p-value	Statistic Z	p-value
Furnas	-0.405	0.687 (S)	0.722	0.365 (S)	-0.335	0.738 (S)
Água Vermelha	-0.342	0.734 (S)	0.876	0.697 (S)	-0.438	0.661 (S)
Nova Ponte	0.939	0.352 (S)	0.844	0.623 (S)	1.261	0.207 (S)
Emborcação	1.731	0.089 (S)	1.292	0.512 (S)	1.922	0.055 (S)
São Simão	0.393	0.695 (S)	1.574	0.225 (S)	0.205	0.837 (S)
Barra Bonita	-1.138	0.261 (S)	0.851	0.641 (S)	-1.112	0.266 (S)

Promissão	-1.927	0.058 (S)	0.709	0.348 (S)	-1.599	0.110 (S)
Jupiá	-0.923	0.360 (S)	1.427	0.332 (S)	-0.941	0.347 (S)
Jurumirim	-2.191	0.037 (NS)	0.315	0.001 (NS)	-2.121	0.034 (NS)
Capivara	-2.274	0.027 (NS)	0.316	0.002 (NS)	-1.725	0.084 (S)

NOTE: (S) - Stationary; (NS) – Non-Stationary

Table 4 - Results of Spearman’s Rho, Cox-Stuart and Run tests.

HPP	**SPEARMAN´S RHO TEST**		**COX-STUART TEST**		**RUN TEST**	
	Statistic Z	**p-value**	**Statistic S**	**p-value**	**Statistic R**	**p-value**
Furnas	1.093	0.274 (S)	15	0.860 (S)	25	0.034 (NS)
Água Vermelha	1.197	0.231 (S)	11	0.110 (S)	25	0.034 (NS)
Nova Ponte	-0.839	0.401 (S)	12	0.215 (S)	35	0.694 (S)
Emborcação	-1.504	0.133 (S)	13	0.377 (S)	32	0.720 (S)
São Simão	-0.030	0.976 (S)	13	0.377 (S)	30	0.382 (S)
Barra Bonita	1.733	0.083 (S)	12	0.215 (S)	36	0.531 (S)
Promissão	1.951	0.051 (S)	13	0.377 (S)	36	0.531 (S)
Jupiá	1.388	0.165 (S)	13	0.377 (S)	29	0.261 (S)
Jurumirim	2.609	0.009 (NS)	7	0.002 (NS)	30	0.382 (S)
Capivara	2.343	0.019 (NS)	10	0.050 (NS)	22	0.004 (NS)

NOTE: (S) - Stationary; (NS) – Non-Stationary

Table 5 - Results of Mann-Kendall and multivariate Mann-Kendall tests.

HPP	**MANN-KENDALL TEST**		**MULTIVARIATE MANN-KENDALL TEST**	
	Statistic Z	**p-value**	**Statistic Z**	**p-value**
Furnas	1.246	0.213 (S)	1.438	0.150 (S)
Água Vermelha	1.353	0.176 (S)		
Nova Ponte	-0.753	0.451 (S)		
Emborcação	-1.432	0.152 (S)		
São Simão	-0.028	0.977 (S)		
Barra Bonita	1.693	0.091 (S)		
Promissão	1.976	0.048 (NS)		
Jupiá	1.455	0.146 (S)		
Jurumirim	2.599	0.009 (NS)		
Capivara	2.282	0.023 (NS)		

NOTE: (S) - Stationary; (NS) – Non-Stationary

Table 6 shows the results of the likelihood ratio tests for the selection of the extreme values distribution (GEV or Gumbel) in the stationary model of each HPP.

Table 6: Extreme Values Distribution Selection for the stationary model (f10)

HPP	Likelihood ratio	p-value	f10 Model
Furnas	0.832	0.362	GUMBEL
Água Vermelha	6.355	0.012	GEV
Nova Ponte	0.596	0.440	GUMBEL
Emborcação	0.136	0.713	GUMBEL
São Simão	1.975	0.160	GUMBEL
Barra Bonita	1.517	0.218	GUMBEL
Promissão	1.630	0.202	GUMBEL
Jupiá	7.925	0.005	GEV
Jurumirim	5.095	0.024	GEV
Capivara	3.428	0.006	GUMBEL

Considering the selected extreme values distribution for the stationary model at each HPP, specific tests were applied considering different forms of non-stationarity. Table 7 shows for Água Vermelha HPP's time series the parameters estimated by maximum likelihood for the selected distribution of the stationary model, as well as the parameters of the three types of non-stationary model derived from the stationary model. Also shown are the negative log-likelihood of each model, the values of the likelihood ratio test and the associated p-value.

Table 7 – Estimates of parameters and negative log-likelihood for the stationary and non-stationary models of Água Vermelha HPP's series. Values of the Likelihood-Ratio Test and the associated p-value.

ÁGUA VERMELHA - Selected extreme values distribution: GEV								**Likelihood ratio (p-value)**
Model		**μ_0**	**μ_1 (t)**	**σ_0**	**σ_1 (t)**	**ξ**	**- Log (V)**	
Stationary (f10)	Parameter estimate	0.891		0.283		-0.235	11.707	x
	Standard error estimate	0.039		0.027		0.080		
Location varying in time (f11)	Parameter estimate	0.893	0.032	0.282		-0.241	11.285	**f10 x f11** 0,485 (0,358)
	Standard error estimate	0.039	0.035	0.027		0.077		
Scale varying in time (f12)	Parameter estimate	0.888		0.283	0.015	-0.237	11.553	**f10 x f12** 0,310 (0,578)

	Standard error estimate	0.039		0.027	0.027	0.079		
Location and scale varying in time (f13)	Parameter estimate	0.893	0.044	0.281	0.026	-0.239	10.842	x
	Standard error estimate	0.039	0.037	0.027	0.027	0.078		

Table 8 presents for each HPP the model chosen by the proposed procedure, as well as the estimates of its parameters and standard errors (CHAN *et al.*, 2019-1 and CHAN *at al.*, 2019-2 show detailed results for each HPP).

Table 8 – Chosen model, estimates of parameters and their standard errors.

HPP	Model		μ_0	μ_1 (t)	σ_0	σ_1 (t)	ξ
Furnas	f10	Parameter estimate	0.844		0.280		
		Standard error estimate	0.037		0.027		
Água Vermelha	f10	Parameter estimate	0.891		0.283		-0.235
		Standard error estimate	0.039		0.027		0.080
Nova Ponte	f10	Parameter estimate	0.830		0.306		
		Standard error estimate	0.040		0.029		
Emborcação	f10	Parameter estimate	0.832		0.294		
		Standard error estimate	0.038		0.028		
São Simão	f10	Parameter estimate	0.856		0.264		
		Standard error estimate	0.035		0.024		
Barra Bonita	f10	Parameter estimate	0.819		0.329		
		Standard error estimate	0.043		0.031		
Promissão	f10	Parameter estimate	0.821		0.325		
		Standard error estimate	0.043		0.031		
Jupiá	f10	Parameter estimate	0.902		0.252		-0.218
		Standard error estimate	0.034		0.023		0.060
Jurumirim	f13	Parameter estimate	0.797	0.132	0.303	0.081	0.080
		Standard error estimate	0.047	0.042	0.039	0.039	0.127
Capivara	f13	Parameter estimate	0.792	0.160	0.353	0.100	
		Standard error estimate	0.048	0.046	0.038	0.042	

5. Conclusion

The results of the seven general statistical test selected for this study summarized in Tables 3, 4 and 5 suggest a stationary behaviour for the time series of annual maximum streamflow of 10-days averages to the HPPs Nova Ponte, Emborcação, São Simão, Barra Bonita and Jupiá. In the case of the HPPs Furnas, Água Vermelha and Promissão time series, among the seven tests, only one rejected the hypothesis of stationarity, also indicating stationary behaviour for these time series. On the other hand, the hypothesis of stationarity of the time series of the HPPs Jurumirim and Capivara were rejected in six of the seven selected tests, which suggests a non-stationary behaviour.

The results obtained with the application of the specific tests for extreme value s indicated stationary behaviour for 8 of the 10 series. Only the series of the HPPs Jurumirim and Capivara indicated non-stationary behaviour, in both cases the best model consider location and scale parameters varying linearly in time.

Among the ten time series selected for the case study, four of them were also selected for the study described in (DAMÁZIO and COSTA, 2014). The conclusions for three of these time series (HPPs Furnas, Barra Bonita and Jupiá) were divergent as to the stationary behaviour over time. Although both analyses were made considering the distributions of extremes, these divergences can be the consequences of two factors: (i) the type of maximum analysed and (ii) the period analysed. In this work, the series of maximum annual flow of 10-day averages for the period 1951 to 2015 were analysed, while DAMÁZIO and COSTA (2014) analysed the series of maximum annual daily streamflow, which were determined as the maximum value presented in the daily series for the hydrological year, obtained for the period from 1931 to 2013.

The next step of this work will focus on investigating the reason of the non-stationary behaviour of the time series of maximum annual flows of 10-days averages of the HPPs Jurumirim and Capivara.

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