CADERNOS DO IME – Série Estatística
Universidade do Estado do Rio de Janeiro - UERJ
ISSN impresso 1413-9022 / ISSN on-line 2317-4536 - v.42, p.01 -16, 2017
DOI: 10.12957/cadest.2017.25564

THE EFFECT OF MISCLASSIFICATION DUE TO MEASUREMENT ERROR ON CUSUM CONTROL CHARTS FOR INTERVENED POISSON DISTRIBUTION

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Abstract

In this paper the one-sided CUSUM chart for controlling the incidence and intervention parameters of the IPD under misclassification error due to measurement is discussed. Explicit formulae are derived for this purpose. The sensitivity of the parameters of the V-mask and the Average Run Length (ARL) is studied through numerical evaluation for grid of values. Numerical results presented reveal that the angle ϕ of the mask increases slightly as shift in the ratio $\frac{\theta_{e_1}}{\theta_{e_0}}$ decreases, whereas, for fixed α, the values of d decrease considerably as the deviation of θ_{e_0} from θ_{e_1} increases. It is also shown that measurement error lessens the consumer's risk, e_2 (because it gives early detection for the shift of the process parameter) and increases the producer's risk, e_1. Further for fixed e_1, e_2, α, θ_{e_0}, θ_{e_1} the values of ARL decrease as we increase the values of ρ.

Keywords: *Measurement error, Misclassification, Intervened Poisson distribution.*

1. Introduction

Quality control charts, one type of the tools in quality management and critical aid to the quality control (DOU and PING, 2002) are widely employed to monitor and ensure the process stability. The strength of control charts lies in their ability to reveal process shifts and identify abnormal conditions in the production process.

One of the most popular control charts in industry is the cumulative sum (CUSUM) control chart which can be employed both when the quality characteristic is a continuous random variable (for monitoring the mean and variance) and when it is a discrete attribute (NENES & TAGARAS, 2010). Johnson and Leone (1962) developed mathematical procedures for construction of the CUSUM control chart for Poisson variable using the relationship between Wald's Sequential Probability Ratio Test (SPRT) and the CUSUM. Lucas (1985) illustrated the design and implementation procedure for counted data for detection of increase or decrease in the count level. A comprehensive overview of CUSUM charts for various probability distributions is given by Hawkins and Olwell (1998). We refer any interested reader to Qiu (2014) for a related discussion.

The design of a CUSUM chart assumes that the procedure adopted is error-free. In practice, inspection procedures are not always perfect and are usually subject to errors. Error effect on control charts have generated much more interest and a growing body of literature on the issue is also available. Singh (1964), Kanazuka (1986), Singh and Sayyed (2001), Singh *et al.* (2002), Balamurali and Kalyanasundaram (2011), Maravelakis (2012), Sankle and Singh (2012), Sankle *et al.* (2012) and Chakraborty and Khurshid (2013 a, b) and references therein have studied the nature and magnitude of measurement error and its effect on the actual performance of various control charts. Recently, Chakraborty and Khurshid (2016) studied the effect of misclassification on the power of a control chart for proportions. The Poisson distribution plays a major role in any given statistical quality control process. However on many occasions, probability distributions often arise which are of the Poisson type but in which the zero value is unobserved. This is the Zero-Truncated Poisson Distribution (ZTPD). For example, the number of occupants in a bus on the road can be modeled by a ZTPD because buses on the road with zero occupants cannot be monitored. For concrete examples Best *at al.* (2007) may be referred to. Chakraborty and Kakoty (1987) constructed a CUSUM scheme for ZTPD.

Another type of probability distribution has received much attention in the literature in which the notion of intervention has been incorporated. These types of distributions furnish evidence on the usefulness of numerous preventive actions discussed in many areas of scientific research. Shanmugam (1985) presented an intervened Poisson distribution (IPD) as an alternate for ZTPD where certain intervention process alters the mean of rare events. The IPD has been widely used, primarily in epidemiological studies, reliability settings and queuing problems and has been further studied by Shanmugam (1992); Huang and Fung (1989) and Dhanavanthan (1998, 2000).

The objective of the present paper is to investigate the effect of misclassification due to measurement error on power of control chart for IPD. Separate formulae are derived for calculating probabilities of misclassification due to measurement of error for incidence and intervention parameters of the IPD. The article is structured as follows. Section 2 introduces IPD and lists some known results. Section 3 briefly discusses about misclassification error and Lavin's expression is presented. In Section 4 we develop a procedure and construct a one-sided CUSUM chart for controlling the incidence and intervention parameters of the IPD under misclassification error due to measurement. To explore the sensitivity of the monitoring procedure, average run length for both the parameters are also derived in this section. Section 5 presents numerical results over a grid of values and some discussions are made and conclusions in Section 6 close the paper.

2. Intervened Poisson distribution (IPD)

A modified version of ZTPD, which is called an Intervened Poisson Distribution (IPD) as considered by Shanmugam (1985) was introduced in medicine in the context of cholera cases: Let Y be the number of cholera cases per household where the event $Y=0$ is unobservable and assume that the distribution of Y is a ZTPD with parameter θ. Assuming that a new preventive measure alters θ to $\theta\rho$, $0 \le \rho \le \theta$. Let Z be the total number of cholera cases that occurred after the preventive measure is applied and assume that Z is a Poisson $(\rho\theta)$ random variable. Assuming that Y and Z are stochastically independent and $X = Y + Z$ represent the total number of cholera cases then the resulting distribution of $X = Y + Z$ is given by following probability mass function

$$P(X = x) = \frac{[(1+\rho)^x - \rho^x]}{[e^{\rho\theta}(e^{\theta} - 1)]} \frac{\theta^x}{x!} \tag{2.1}$$

where $x = 1,2,...,$ $\theta > 0,$ and $0 \le \rho \le \infty$. Equation (2.1) is commonly called IPD identified by Shanmugam (1985). Here θ and ρ are called incidence and intervention parameters respectively. The mean and variance of IPD with probability mass function (2.1) are, respectively

$$E(X) = \theta[\rho + 1 + (e^{\theta} - 1)^{-1}], \tag{2.2}$$

$$Var(X) = \theta[\rho + 1 + (e^{\theta} - 1)^{-1}] - e^{\theta}\left(\frac{\theta}{e^{\theta} - 1}\right)^2 \tag{2.3}$$

We observe that when $\rho = 0$ we get the ZTPD.

In the last decades, there has been considerable interest in intervened distributions and their variants (see, for example, DHANAVANTHAN, 1998, 2000; HUANG & FUNG, 1989; KUMAR & SHIBU, 2011, 2012, 2013; KUMAR & SREEJAKUMARI, 2012, 2017; PATEL, 1999; PATEL & GAJJAR, 1990, 2000; SCOLLNIK, 1995, 2006). Much of this interest stems from the pioneering paper of Shanmugam (1985), though this type of model appears to have originated in the field of medicine. The main advantage of IPD is that it provides information on the effectiveness of various preventive actions (generally taken by health service agencies etc.) whereas ZTPD does not. Applications of IPD in various fields are illustrated in Shanmugam (1985, 1992, 2005), Johnson *et al*. (2005). Kakoty and Chakraborty (1990) studied CUSUM control chart for IPD to control the incidence and intervention parameters of the IPD.

3. Misclassification error and expression for the true and apparent non-conformities

One important way of judging the performance of any classification procedure is to calculate its error (type I and type II) rates or misclassification probabilities. In every inspection system, there may be either of two possible types of errors: (i) a good (conforming) item to a specification may be misclassified as defective (non-conforming) or (ii) a defective (non-conforming) item may be misclassified as good (conforming). These types of errors are recognized as errors due to misclassification and are generally due to chance causes.

Thus, if e_1 and e_2 are type I and type II errors, and θ is the true nonconformities, then the relation between true nonconformities θ and apparent (observed) nonconformities θ_e is given by Lavin (1946) by his equation as (see also COLLINS & CASE, 1976; COLLINS *et al.* 1973; MITTAG & RINNE, 1993 for details)

$$\theta_e = \theta(1-e_2) + e_1(1-\theta). \tag{3.1}$$

For the evaluation technique of e_1, e_2 and θ_e, one may refer to Chakraborty and Khurshid (2016).

4. CUSUM control chart (for the control of incidence parameter)

Let $x_1, x_2, ..., x_m$ be i.i.d. random variables each distributed with probability mass function (2.1). To test the null hypothesis $H_0 : \theta_e = \theta_{e_0}$ against the alternative hypothesis $H_1 : \theta_{e_1} (> \theta_{e_0})$, (under misclassification error) assuming ρ known, following Johnson (1961), we use the likelihood ratio

$$\frac{f(x_1, x_2, ..., x_m \,|\, \theta_{e_1}, \rho)}{f(x_1, x_2, ..., x_m \,|\, \theta_{e_0}, \rho)} = \left(\frac{\theta_{e_1}}{\theta_{e_0}}\right)^{\sum_{i=1}^{m} x_i} \left[e^{\rho(\theta_{e_0} - \theta_{e_1})} \left(\frac{e^{\theta_{e_0}} - 1}{e^{\theta_{e_1}} - 1} \right) \right]. \tag{4.1}$$

The continuation region of the SPRT discriminating between the two hypotheses is given by

$$\log\left(\frac{\beta}{1-\alpha}\right) < \log\left(\frac{\theta_{e_1}}{\theta_{e_0}}\right) \sum_{i=1}^{m} x_i + m\left[\rho(\theta_{e_0} - \theta_{e_1}) + \log\left(\frac{e^{\theta_{e_0}} - 1}{e^{\theta_{e_1}} - 1} \right) \right] < \log\left(\frac{1-\beta}{\alpha}\right) \tag{4.2}$$

where α and β are the probabilities of type I and type II errors respectively.

For a very small value of β, we obtain that the right hand side inequality of Equation (4.2) reduces to

$$\sum_{i=1}^{m} x_i < \frac{-\log\alpha - m\left[\rho(\theta_{e_0}-\theta_{e_1}) + \log\left(\frac{e^{\theta_{e_0}}-1}{e^{\theta_{e_1}}-1}\right)\right]}{\log\left(\frac{\theta_{e_1}}{\theta_{e_0}}\right)}. \tag{4.3}$$

For constructing the CUSUM chart we plot the sum $S_m = \sum_{i=1}^{m} x_i$ against the number of observations m. Suppose O is the last plotted point, P is the vertex of the mask and the point Q is obtained by drawing a perpendicular to the line OP. The change in the value of θ_e from θ_{e_0} to θ_{e_1} is detected if any plotted point falls below the line PQ. In this case the parameters of the mask, namely the lead distance $d = OP$ and the angle of the mask $\phi = \angle OPQ$ are given by

$$d = -\log\alpha\left[\log\left(\frac{\exp(\theta_{e_1})-1}{\exp(\theta_{e_0})-1}\right) - \rho(\theta_{e_0}-\theta_{e_1})\right]^{-1} \tag{4.4}$$

and

$$\phi = \tan^{-1}\left[\left\{\log\left(\frac{\exp(\theta_{e_1})-1}{\exp(\theta_{e_0})-1}\right) - \rho(\theta_{e_0}-\theta_{e_1})\right\}\Big/\log\left(\frac{\theta_{e_1}}{\theta_{e_0}}\right)\right]. \tag{4.5}$$

To identify the nature of the parameters of the V-mask we differentiate d and ϕ partially with respect to θ_{e_1}. Thus from Equation (4.4), we have

$$\frac{\partial d}{\partial\theta_{e_1}} = \frac{\log\alpha\left[\left\{\frac{\exp(\theta_{e_1})}{\exp(\theta_{e_1})-1}\right\}+\rho\right]}{\left[\log\left(\frac{\exp(\theta_{e_1})-1}{\exp(\theta_{e_0})-1}\right) - \rho(\theta_{e_0}-\theta_{e_1})\right]^2}\ . \tag{4.6}$$

This will be negative, if $\left[\left\{\frac{\exp(\theta_{e_1})}{\exp(\theta_{e_1})-1}\right\}+\rho\right] > 0$, suggesting that the distance decreases as θ_{e_1} increases. The maximum or minimum of the distance d is achieved by solving $\frac{\partial d}{\partial\theta_{e_1}} = 0$ which gives $\theta_{e_1} = \log\left(\frac{\rho}{\rho-1}\right)$.

Again differentiating Equation (4.6), partially with respect to θ_{e_1} gives

$$\frac{\partial^2 d}{\partial\theta_{e_1}^2}=\frac{-\log\alpha\left[\log\left(\dfrac{\exp(\theta_{e_1})-1}{\exp(\theta_{e_0})-1}\right)-\rho(\theta_{e_0}-\theta_{e_1})+2\left(\exp(\theta_{e_1})+\rho(\exp(\theta_{e_1})-1)^2\right)\right]}{\left(\exp(\theta_{e_1})-1\right)^2\left[\log\left(\dfrac{\exp(\theta_{e_1})-1}{\exp(\theta_{e_0})-1}\right)-\rho(\theta_{e_0}-\theta_{e_1})\right]^3}.$$

(4.7)

Thus, d is maximum (minimum) at $\theta_{e_1}=\log\left(\dfrac{\rho}{\rho-1}\right)$, iff $\dfrac{\partial^2 d}{\partial\theta_{e_1}^2}$ is negative (positive).

Similarly for the angle of the mask, we get from Equation (4.5)

$$\frac{\partial\phi}{\partial\theta_{e_1}}=\frac{\log\left(\dfrac{\theta_{e_1}}{\theta_{e_0}}\right)\left(\dfrac{\exp(\theta_{e_1})+\rho(\exp(\theta_{e_1})-1)}{\exp(\theta_{e_0})-1}\right)-\left[\dfrac{\log\{(\exp(\theta_{e_1})-1)/(\exp(\theta_{e_1})-1)\}+\rho(\theta_{e_1}-\theta_{e_0})}{\theta_{e_1}}\right]}{\left[\log\left(\dfrac{\theta_{e_1}}{\theta_{e_0}}\right)\right]^2+\log\left(\dfrac{\exp(\theta_{e_1})-1}{\exp(\theta_{e_0})-1}\right)+\rho(\theta_{e_1}-\theta_{e_0})}.$$

. (4.8)

If $\dfrac{\partial\phi}{\partial\theta_{e_1}}>0$ then the angle of the mask increases as θ_{e_1} increases or decreases. On the other hand if Equation (4.8) is negative, the ϕ increases as θ_{e_1} decreases and vice-versa. The maximum or minimum is obtained by differentiating Equation (4.8) with respect to θ_{e_1}.

4.1. Average run Length (ARL) under misclassification error

Following Johnson (1961), the approximate formula for ARL for detecting a shift in the mean value θ_e from θ_{e_0} to θ_{e_1} is given by $ARL=(-\log\alpha)E_{\theta_{e_1}}^{-1}$ where $E_{\theta_{e_1}}^{-1}=E\left[\log\dfrac{f(x|\theta_{e_1})}{f(x|\theta_{e_0})}\middle|\theta_{e_1}\right]$. Thus, from Kakoty and Chakaraborty (1990), we have

$$ARL=-\log\alpha\left[\rho(\theta_{e_0}-\theta_{e_1})+\log\left(\frac{\exp(\theta_{e_0})-1}{\exp(\theta_{e_1})-1}\right)+\left\{\theta_{e_1}(1+\rho)+\theta_{e_1}(\exp(\theta_{e_1})-1)^{-1}\right\}+\log\left(\frac{\theta_{e_1}}{\theta_{e_0}}\right)\right]^{-1}.$$

(4.9)

To find the characteristic of ARL, we differentiate Equation (4.9) with respect to θ_{e_1}, which produces

$$\frac{\partial ARL}{\partial \theta_{e_1}} = \log \alpha \cdot (A+B) \cdot C \qquad (4.10)$$

where

$$A = \frac{-\rho(\exp(\theta_{e_1})-1)-\exp(\theta_{e_1})}{\exp(\theta_{e_1})},$$

$$B = \left[\frac{\rho(\exp(\theta_{e_1})-1)^2 + \exp(\theta_{e_1})(\exp(\theta_{e_1})-1) + \exp(\theta_{e_1}) - 1 + \theta_{e_1}(\exp(\theta_{e_1}))}{(\exp(\theta_{e_1})-1)} \frac{1}{\theta_{e_1}} \right]$$

and

$$C = \left[\rho(\theta_{e_0} - \theta_{e_1}) + \log\left(\frac{\exp(\theta_{e_0})-1}{\exp(\theta_{e_1})-1} \right) + \left\{ \rho\theta_{e_1} + \theta_{e_1} + \theta_{e_1}\left(\exp(\theta_{e_1})-1\right)^{-1} \right\} \log\left(\frac{\theta_{e_1}}{\theta_{e_0}} \right) \right]^{-2}.$$

Equation (4.10) will be negative if the numerator is less than zero, implying that the ARL decreases as θ_{e_1} increases. Tables 1, 2 and 3 depict some numerical values of d, ϕ and ARL for a number of combinations of the values of α, θ_{e_0}, θ_{e_1}, ρ for some values of e_1 and e_2.

4.2. Control of intervention parameter under misclassification error for the CUSUM control chart of IPD

In this section, we study the problem of controlling the parameter ρ assuming that incidence parameter θ is known. Thus, under misclassification error, for detecting change in the value of ρ_e from ρ_{e_0} to $\rho_{e_1} (> \rho_{e_0})$, the approximate likelihood ratio is

$$\frac{f(x_1, x_2, ..., x_m | \theta, \rho_{e_1})}{f(x_1, x_2, ..., x_m | \theta, \rho_{e_0})} = \exp\{(m\theta)(\rho_{e_0} - \rho_{e_1})\} \left[\frac{(1-\rho_{e_1})^{\sum_{i=1}^{m}(x_i - 1)}}{(1-\rho_{e_0})^{\sum_{i=1}^{m}(x_i - 1)}} \right]. \qquad (4.11)$$

The change in the value of ρ_e will be detected if the inequality $\sum_{i=1}^{m}(x_i - 1) > \dfrac{-\log \alpha + (m\theta)(\rho_{e_1} - \rho_{e_0})}{\log\left[\dfrac{1-\rho_{e_1}}{1-\rho_{e_0}}\right]}$ holds good.

To construct the CUSUM chart in this case also we plot the sum $S_m = \sum_{i=1}^{m}(x_i - 1)$ against the number of observations m. The change of ρ_e from ρ_{e_0} to ρ_{e_1}, is detected if any plotted points fall below the line PQ. The parameters of the mask (the lead distance, d and the angle, ϕ) are given by

$$d = -\log\alpha[\theta(\rho_{e_1} - \rho_{e_0})]^{-1} \tag{4.12}$$

and

$$\phi = \tan^{-1}\left[\frac{\theta(\rho_{e_1} - \rho_{e_0})}{\log\left(\dfrac{1-\rho_{e_1}}{1-\rho_{e_0}}\right)}\right]. \tag{4.13}$$

4.3. Average Run Length (ARL)

The expected number of observations for detecting a shift in ρ_e from ρ_{e_0} to ρ_{e_1} is approximately given by

$$ARL = -\log\alpha\left[-\theta(\rho_{e_1} - \rho_{e_0}) - \log\left(\frac{1-\rho_{e_1}}{1-\rho_{e_0}}\right) + \left\{\theta(\rho_{e_1} + 1) + \theta\left(\exp(\theta) - 1\right)^{-1}\right\}\log\left(\frac{1-\rho_{e_1}}{1-\rho_{e_0}}\right)\right]^{-1}, \tag{4.14}$$

assuming that incidence parameter θ is known. In this case also the same theoretical operation can be applied to understand the nature of the parameters of the V-mask and the ARL.

5. Numerical results

Table 1, using Equation (4.4), which gives values of d for controlling the incidence parameter for a number of combinations of α, θ_{e_0}, θ_{e_1}, ρ for some values of e_1 and e_2 shows that for a fixed α, the values of d decreases considerably as the difference $(\theta_{e_1} - \theta_{e_0})$ increases, whereas for the same difference $(\theta_{e_1} - \theta_{e_0})$ it increases as α decreases for fixed e_1, e_2 and ρ.

To see the error effect (error due to misclassification), it has been observed from Table 1 (A to E) that the values of d increase considerably as we increase e_1

for fixed e_2 and ρ when the ratio between ($\frac{\theta_{e_1}}{\theta_{e_0}}$) decreases, whereas for fixed e_1 and ρ the values of d increase considerably as there is an increase in e_2 when the corresponding ratio ($\frac{\theta_{e_1}}{\theta_{e_0}}$) increases. But for fixed e_1, e_2 the values of d decrease as we increase the values of intervention parameter ρ for the fixed difference. It is interesting to note that the values of d are always less when $e_1 = e_2 = 0$ for fixed ρ and fixed difference.

Table 1: Values of d for testing incidence parameter (using Equation (4.4))

(a) When $e_1 = 0$, $e_2 = 0.30$ and $\rho = 2$

θ_0	θ_{e_0}	θ_1	θ_{e_1}	α				
				0.05	0.025	0.01	0.005	0.001
0.5	0.35	1	0.7	1.8921	2.3298	2.9085	3.3463	4.3627
		2	1.4	0.7331	0.9027	1.1269	1.2965	1.6904
		3	2.1	0.4726	0.5821	0.7267	0.8361	1.0900
		4	2.8	0.3521	0.4336	0.5413	0.6228	0.8120
1	0.7	2	1.4	1.1977	1.4737	1.8397	2.1166	2.7596
		3	2.1	0.6299	0.7757	0.9683	1.1141	1.4525
		4	2.8	0.4327	0.5328	0.6651	0.7653	0.9977

(b) When $e_1 = 0.02$, $e_2 = 0$ and $\rho = 2$

θ_0	θ_{e_0}	θ_1	θ_{e_1}	α				
				0.05	0.025	0.01	0.005	0.001
0.5	0.51	1	1	1.5531	1.9125	2.3875	2.7469	3.5813
1	0.7	2	1.98	0.5784	0.7129	0.8892	1.0230	1.3338
		3	2.98	0.3620	0.4458	0.5565	0.6402	0.8843
		4	3.98	0.2649	0.3262	0.4072	0.4685	0.6108

(c) When $e_1 = 0.02$, $e_2 = 0.30$ and $\rho = 2$

θ_0	θ_{e_0}	θ_1	θ_{e_1}	α				
				0.05	0.025	0.01	0.005	0.001
0.5	0.36	1	0.7	1.9581	2.4112	3.0101	3.4631	4.5151
		2	1.38	0.7553	0.9300	1.1610	1.3358	1.7415
		3	2.06	0.4863	0.5989	0.7476	0.8601	1.1214
		4	2.74	0.3623	0.4461	0.5569	0.6407	0.8353

(d) When $e_1 = 0.02$, $e_2 = 0.30$ and $\rho = 3$

θ_0	θ_{e_0}	θ_1	θ_{e_1}	α				
				0.05	0.025	0.01	0.005	0.001
0.5	0.36	1	0.70	1.6021	1.9728	2.4628	2.8335	3.6942
		2	1.38	0.6008	0.7398	0.9235	1.0625	1.3853
		3	2.06	0.3811	0.4693	0.5859	0.6741	0.8789
		4	2.74	0.2813	0.3464	0.4324	0.4975	0.6486

		(e) When $e_1 = 0$, $e_2 = 0$ **and** $\rho = 2$				
θ_0	θ_1	α				
		0.05	0.025	0.01	0.005	0.001
0.5	1	1.5176	1.8686	2.2328	2.6816	3.4992
	2	0.5666	0.6977	0.8710	1.0021	1.3065
	3	0.3574	0.4401	0.5494	0.6321	0.8242
1	2	0.9042	1.1134	1.3899	1.5991	2.0849
	3	0.4631	0.5758	0.7188	0.8269	1.0782
	4	0.3167	0.3900	0.4869	0.5602	0.7303

Table 2, employing Equation (4.5), shows that the angle ϕ of the mask increases as the ratio $\frac{\theta_{e_1}}{\theta_{e_0}}$ decreases. It is also evident from Table 2 that for fixed e_1, the angle of the mask decreases as we increase the values of e_2 for fixed ρ when the ratio between ($\frac{\theta_{e_1}}{\theta_{e_0}}$) decreases. But for fixed ratio, e_1 and e_2 the angle of the mask increases as we increase the values of ρ. In the absence of misclassification error i. e., $e_1 = e_2 = 0$, the angle of the V-mask is always greater for a fixed intervention parameter.

Table 2: Values of ϕ (degree) for controlling incidence parameter (employing Equation (4.5))

	(a) When $e_1 = 0$, $e_2 = 0.30$ **and** $\rho = 2$			
	θ_{e_1}			
θ_{e_0}	0.70	1.4	2.1	2.8
0.35	66.36	71.26	74.22	76.26
0.70	-	74.52	76.99	78.68

	(b) When $e_1 = 0.02$, $e_2 = 0$ **and** $\rho = 2$			
	θ_{e_1}			
θ_{e_0}	1	1.98	2.98	3.98
0.51	70.76	75.32	77.96	79.70

	(c) When $e_1 = 0.02$, $e_2 = 0.30$ **and** $\rho = 2$			
	θ_{e_1}			
θ_{e_0}	0.7	1.38	2.06	2.74
0.36	66.51	71.29	74.19	76.21

	(d) When $e_1 = 0.02$, $e_2 = 0.30$ and $\rho = 3$			
	θ_{e_1}			
θ_{e_0}	0.7	1.38	2.06	2.74
0.36	70.42	74.92	77.49	79.21

	(e) When $e_1 = 0$, $e_2 = 0$ and $\rho = 2$				
	θ_{e_1}				
θ_{e_0}	1	2	3	4	5
0.5	72.06	75.31	78.00	79.69	80.93
1.5	-	79.85	81.61	82.79	83.61
2.5	-	-	83.23	84.12	84.77

Table 3, utilizing Equation (4.9), shows the values of ARL for different combinations of e_1, e_2, α, θ_{e_0}, θ_{e_1}, ρ. It is seen from Table 3 that the ARL changes in the same direction as the values of d for e_1, e_2 and ρ**.** It is observed from the table that the values of ARL increase considerably as we increase e_1 for fixed α, e_2 and ρ when the values of the ratio ($\frac{\theta_{e_1}}{\theta_{e_0}}$) decrease. A similar trend is observed when the values of e_2 increase for fixed e_1 but the magnitude of the difference in the ARL value is higher when e_2 increases for fixed e_1 (which can be observed if we compare Table 3 (A and C with B and C)). But for fixed e_1, e_2, α, θ_{e_0}, θ_{e_1} the values of ARL decrease as we increase the values of ρ.

Table 3: Values of ARL for controlling incidence parameter (utilizing Equation (4.9))

(a) When $e_1 = 0$, $e_2 = 0.30$ and $\rho = 2$								
θ_0	θ_{e_0}	θ_1	θ_{e_1}	α				
				0.05	0.025	0.01	0.005	0.001
0.5	0.35	1	0.7	8.5423	10.519	13.13	15.11	19.70
		2	1.4	1.2635	1.5558	1.9422	2.2346	2.9134
		3	2.1	0.5472	0.6739	0.8412	0.9679	1.2619
		4	2.8	0.3208	0.3951	0.4932	0.5875	0.7398

(b) When $e_1 = 0.02$, $e_2 = 0$ and $\rho = 2$								
θ_0	θ_{e_0}	θ_1	θ_{e_1}	α				
				0.05	0.025	0.01	0.005	0.001
0.5	0.51	1	1	6.2041	7.6396	9.5373	10.973	14.306
		2	1.98	0.9119	1.1229	1.4018	1.6128	2.1027

		3	2.98	0.3847	0.5913	0.7382	0.8493	1.1073
		4	3.98	0.2239	0.2757	0.3442	0.3960	0.5163

(c) When $e_1 = 0.02$, $e_2 = 0$ **and** $\rho = 2$								
θ_0	θ_{e_0}	θ_1	θ_{e_1}	α				
				0.05	0.025	0.01	0.005	0.001
0.5	0.36	1	0.7	9.1955	11.323	14.136	16.263	21.204
		2	1.38	1.3496	1.6618	2.7046	2.3869	3.1119
		3	2.06	0.5822	0.7169	0.8950	1.0297	1.3425
		4	2.74	0.3405	0.4193	0.5235	0.6022	0.7852

(d) When $e_1 = 0.02$, $e_2 = 0.3$ **and** $\rho = 3$								
θ_0	θ_{e_0}	θ_1	θ_{e_1}	α				
				0.05	0.025	0.01	0.005	0.001
0.5	0.36	1	0.7	6.6383	8.1742	10.205	11.741	15.307
		2	1.38	0.9809	1.2079	1.5079	1.7348	2.2618
		3	2.06	0.4256	0.5241	0.6543	0.7527	0.9814
		4	2.74	0.2501	0.3080	0.3845	0.4423	0.5767

Note: θ_0 and θ_1 are incidence parameters under null and alternative hypotheses.

Similar calculations can be done and conclusions can be drawn accordingly for controlling the intervention parameter ρ of IPD under the CUSUM scheme (with error due to misclassification).

6. Conclusions

This study presents explicit formulae to construct a one-sided CUSUM chart for controlling the incidence and intervention parameters of the IPD under misclassification error due to measurement. To explore the sensitivity of the monitoring procedure, average run length for both the parameters are also derived. Numerical results presented in Section 5 reveal that the angle ϕ of the mask increases slightly as shift in the ratio $\frac{\theta_{e_1}}{\theta_{e_0}}$ decreases. On the other hand, for fixed α , the values of d decrease considerably as the deviation of θ_{e_0} from θ_{e_1} increases. The result clearly shows that measurement error lessens the consumer's risk, e_2 and increases the producer's risk, e_1 .

Acknowledgments

The authors would like to thank the Editor Professor José Fabiano da Serra Costa and two anonymous referees for careful reading the paper and for their comments which greatly improved the paper.

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